Falling into Traps? Patent Thickets, Patent Commercialization, and Stock Returns*

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Abstract

When firms innovate on the basis of prior patents dispersedly owned by different patent assignees, the fragmented patent ownership results in patent thickets that adversely affect the commercialization of these firms' inventions. We develop a real option model that suggests a negative effect of patent thickets on systematic risk exposure and expected stock returns due to increases in commercialization costs, delays in options exercising, and decreases in future stochastic cash flows. Our empirical analyses based on U.S. patent litigations data, new product announcements data, and public firms' patents and financial data support these model implications.

Keywords. Patent Thicket, Technology Application, Litigation, New Product, Stock Return

JEL Classification. E23, G12, O31, O33

1 Introduction

Human civilization is built on accumulated knowledge generated from past generations' innovative activities; as Sir Isaac Newton remarked, "*If I have seen further it is by standing on the shoulders of giants.*" In particular, the economics and innovation literature has attributed the majority of technological achievements to the combination of pre-existing ideas with new knowledge and techniques.[1] In order to accelerate knowledge accumulation and technical progress, governments introduced patent systems to encourage inventors to share their ideas, discoveries, and inventions with the public in exchange for exclusive usage rights of patented new knowledge for a certain period. By employing effective systematic disclosure and enforcing patent rights, these patent systems allow patentees and producers to negotiate licensing contracts efficiently, as well as generate more innovative products and higher incentives to innovate.

Patentees collect economic rents from their patents through three channels: 1) they can collect royalty from users, including individuals and organizations that negotiate with them to use their patents; 2) they can file, or threaten to file, a patent-infringement lawsuit against individuals or organizations that potentially infringe their patents, in which case patentees collect settlements or royalties if they win the lawsuit or negotiate a deal with or without initiating a lawsuit; or 3) they enjoy market advantages as other individuals and organizations bear monetary and time costs to circumvent their patents. However, for a firm planning to commercialize its patents, the coordination inherent in patent licensing and litigation risk could be costly if related prior patents are dispersedly owned by different patent assignees. This situation is often referred to as a "patent thicket" (Shapiro, 2001; Ziedonis, 2004; Galasso and Schankerman, 2010).[2] Such obstacles usually make any participant in

[1] This recombinant viewpoint is perhaps most famously captured by Joseph A. Schumpeter's regarding innovation as the "carrying out of new combinations" (Schumpeter, 1934, pp. 65-68). Such a viewpoint is also supported by Nelson and Sidney (1982), Weitzman (1998), and Singh and Fleming (2010).

[2] As far as we know, the term "patent thicket" is first used by Shapiro (2001) to refer to an overlapping set of patent rights that require those trying to commercialize new technology to obtain permission from multiple patentees.

a fragmented technology field hesitate to convert patents into products and revenue, due to difficult negotiation with owners of closely related patents and potential infringement risk. Specifically, existent patents appear to form a thicket, which entraps new inventors and prevents them from fully exploiting their new patents. Since patenting firms confronted with deeper patent thickets suffer from lower expected profits and delays in exercising options, we propose that patent thickets adversely affect firm value and have asset pricing implications.[3]

To illustrate the effect of patent thickets on the dynamics of patent exploitation, we first build a tractable partial equilibrium model based on the real option literature.[4] Specifically, we assume that when a firm granted with a patent plans to exercise this option (i.e., transform the patent into an asset in place) and produce stochastic future cash flows, this firm has to pay costs to the owners of other patents surrounding its patent. We show that a deeper patent thicket (i.e., more surrounding patent owners) hinders new technology application and delays option exercise,[5] leading to a lower exposure to systematic risk and a lower expected stock return. We thus form two model predictions related to the cross-sectional variation of stock returns: systematic risk exposure decreases with patent thickets, and so do expected stock returns. The underlying intuition is that a firm encountering a deeper patent thicket receives lower stochastic future cash flows, due to higher patent exploitation costs and option exercise delays; in turn, this firm is then less exposed to systematic risk and generates lower expected returns. In this sense, our model follows a conditional CAPM argument.

[3]In the literature, firms' innovation activities, such as R&D and patenting, are commonly regarded as growth options and largely determine firms' risk exposure and stock returns. Risk exposure and future cash flows of growth options have been studied in the literature, including Berk et al. (1999), Berk et al. (2004), Carlson et al. (2004), Zhang (2005), Aguerrevere (2009), Garleanu et al. (2012), Ai and Kiku (2013), and Kogan and Papanikolaou (2014), among many others.

[4]We assume an exogenously given pricing kernel. One may understandably express concerns that innovation, by its very nature, influences the pricing kernel. However, we argue that such concerns should be minor, for when we focus on cross-sectional firm characteristics (i.e., patent thickets) instead of aggregate shocks, the cash flow effect for a given firm should dominate the discount rate effect. Meanwhile, the advantage of our model is its tractability with a closed-form solution.

[5]The total exploitation cost increases with the number of surrounding patents, due to population and coordination effects. For the population effect, given the same royalty fee per patent, the total royalty cost increases with the number of surrounding patents. For the coordination effect, a larger number of surrounding patents leads to an increase in coordination difficulty and results in higher coordination costs.

We also consider extended models in our Online Appendix. In the basic model, we assume that the patent exploitation costs are determined in a simultaneous bargaining game, in which coordination among all patent owners is muted. To allow for coordination, we consider the sequential bargaining game in an extended model. In addition, while the royalty payment is the only cost related to patent thickets in the basic model, we also consider litigation cost in another extended model. Another assumption in the basic model that a firm is granted with only one patent is also relaxed in another extended model. This extension not only shows that our key implication (namely, patent thickets decrease systematic risk exposure and expected stock returns) still holds, but also delivers new model predictions that are consistent with our empirical results.

In our empirical analysis, we measure patent thicket using the definition of Ziedonis (2004) and the patent data organized by Kogan et al. (forthcoming). Ziedonis' (2004) patent thicket measure is defined as one minus the Herfindahl index based on the shares of other firms' patents being cited by the focal firm's patents that are approved over the past five years.[6] When a focal firm's patents cite prior patents owned by many different owners, this firm is regarded to be confronted with a deeper patent thicket (i.e., more fragmented ownership of related patents). It is noteworthy that applicants to U.S. patents are legally required to provide a comprehensive list of references that includes all prior patents and works that are material to patentability of their applications ("duty of candor and good faith").[7] Given the

[6]This patent thicket measure has been widely used in the industrial organization literature, including Cockburn and MacGarvie (2009), Cockburn et al. (2010), Galasso and Schankerman (2010), Cockburn and MacGarvie (2011), Graevenitz et al. (2013), Noel and Schankerman (2013), among others. The use of a five-year window in measuring patent activities is common in the literature (Rothaermel and Deeds, 2004; Matolcsy and Wyatt, 2008; Hirshleifer, Hsu, and Li, 2013). One may argue that firms' decision to innovate in one technology area depends on prior patent thicket in that area. In our robustness check, we consider an alternative patent thicket measure that is based on a two-year window and is thus unexpected to the focal firm, given that the application-approval lag is around two to three years.

[7]The reference list is subject to patent examiners' reviews. In practice, over 40% of citations are added by patent examiners (Thompson, 2006; Alcacer et al., 2009). If applicants failed to cite relevant patents and works, their applications might be rejected by the USPTO (Caballero and Jaffe, 1993; Roach and Cohen, 2013). When patent applications are approved, the published patent documents will include a full list of prior patents and works based on self-reporting or requests by patent examiners. However, even granted patents may be invalidated by the court later if their reference lists miss important prior arts (Allison and Lemley, 1998; Sampat, 2010).

legal requirement of comprehensive references in the U.S. patent system, we are able to use patent citations to capture closely related patents and measure patent thickets. Moreover, prior studies have shown that patentees tend to initiate an infringement litigation when their patents are cited by other firms that are active in related technology areas (Lanjouw and Schankerman, 2001).

Indeed, we find that a firm's patent thicket measure positively predicts its frequency of being involved in patent litigations in the future five years. In particular, firms in the top quintile of patent thickets will be involved in 0.73 to 1.09 more litigation cases as defendants than their peers in the bottom quintile. Its economic magnitude is considerable as, on average, a sample firm is involved in 0.27 litigation cases in a five-year horizon.[8] Also, its statistical significance is robust to year fixed effects, industry effects, and firm characteristics such as patent portfolio size, the bargaining power of counterparties, R&D intensity, market capitalization, and book-to-market ratio.

Furthermore, we find that a firm's patent thicket measure negatively predicts its ratio of patent commercialization (i.e., the future five-year accumulative number of new products divided by the previous five-year accumulative number of patents granted). Specifically, firms in the top quintile of patent thickets will launch 44% to 115% fewer new products from their patent portfolios than their peers in the bottom quintile, which is statistically significant. As the average ratio of patent commercialization is 31% in a five-year horizon for a sample firm, patent thickets' predictive ability with respect to patent commercialization is substantial.

We then test our Proposition 1: the negative relation between patent thickets and systematic risk exposure. Specifically, we regress firms' market betas or volatilities in firm operations in the next five years on their patent thickets in the current year.[9] In doing so, we obtain the following results. First, if a firm moves from the lowest patent thicket quintile to the highest quintile, its market beta drops by about 0.10 to 0.13 with significance at the

[8] Bessen and Meurer (2012) estimate that when sued for patent infringement, alleged infringers lose about \$28.7 million of market value in 1992 dollars or \$44.6 million in 2010 dollars.

[9] Beaver et al. (1970) and Kothari et al. (2002) show that earnings volatility is positively associated with systematic market risk.

1% level. This number is non-negligable as, on average, the market beta of a sample firm is about 1.24. Second, firms in the lowest patent thicket quintile have significantly higher volatilities in ROA, ROE, investment, and sales compared with their peers in the highest patent thicket quintile.

Finally, to test Proposition 2, we construct portfolio sorting analyses to examine the relation between patent thickets and expected stock returns. We find the following supportive evidence. First, the monthly value-weighted stock returns of firms in the top quintile of patent thickets underperform those of firms in the bottom quintile by 0.46%. This difference is significant at the 1% level. Second, the market beta of the top quintile portfolio is 0.07 lower than that of the bottom quintile portfolio, and such a difference is statistically significant at the 5% level.

To rule out industry heterogeneity, we form industry-balanced portfolios such that all firms in each industry are evenly distributed across the five patent thicket quintile portfolios. When we do so, we find that the relation between patent thickets and stock returns remains. For example, firms in the top quintile of patent thckets underperform their same-industry peers in the bottom quintile by 0.48%. They also tend to have lower market betas.

It is worth noting that our model and empirical analyses assume that patent endowment is independent from patent thicket (i.e., a firm faces an exogenous patent thicket when it receives an endowment of patents). We adopt such a setting because 1) a firm does not know the patent thicket it faces until its own and competitors' patents are all granted, and 2) we focus on the cross-sectional variation of stock returns instead of time-series dynamics. We appreciate that some may question the extent to which our results will change if the value of patents produced by a firm is affected by patent thickets the firm is facing. We conjecture that, in reality, a firm ex ante will only file a patent of sufficiently high value or quality if it expects the cost associated with patent thickets will offset its profit ex post. Such a behavior should lead to a positive relation between patent thickets and systematic risk exposure, as well as with expected stock returns, which is not supported by our empirical evidence.

This study contributes to the literature by highlighting the role of patent thickets in asset prices. Prior studies concerning about innovation and equity pricing, such as Lin (2012), Kogan et al. (forthcoming), Cohen et al. (2013), and Hirshleifer et al. (2013), among others, focus on the value implication of individual firms' technological development.[10] We focus on how the financial market evaluates a firm's patents based on its competitors' prior patents. Our study thus extends the literature by offering a new perspective on financial valuation of technological innovation.

Our paper relates to, yet is distinct from, studies on product market competition. Hou and Robinson (2006) link product quantity competition to stock returns and find that a higher concentration of firm sales leads to lower returns. Hoberg and Phillips (2012) relate product quality competition to stock returns and document that product uniqueness does not have any explanatory power on future stock returns. In exploring the linkage between patent ownership and stock returns, we show that a higher concentration of patent ownership predicts higher stock returns. Specifically, our evidence that the patent thicket effect remains significant–even we control for industry effects–implies that patent thickets comprise a distinct firm characteristic from product market competition. More importantly, our focus on patent portfolios should be more value-relevant than the other two studies' focus on product portfolios because patent portfolios reflect firms' prospects in future competition.

The economics literature has examined the phenomenon of patent thickets, related problems, and possible solutions for some time.[11] With these studies, we now extend the literature by examining the impact of patent thickets from an equity market perspective: we model and empirically verify a relation between patent thickets and firms' market value, systematic risk

[10] Moreover, Greenwood et al. (1997), Hobijn and Jovanovic (2001), Laitner and Stolyarov (2003), Pástor and Veronesi (2009), Hsu (2009), Papanikolaou (2011), and Garleanu et al. (2012) investigate the value implication of technological innovation from an aggregate perspective.

[11] Shapiro (1985 and 2001) and Baron and Pohlmann (2015) investigate whether cross-licensing, patent pools, or cooperative standard setting can mitigate patent thickets. Ziedonis (2004) studies how firms expand their patent portfolios in response to patent thickets. Bessen (2004) considers patent thickets in a strategic patenting context, while Clark and Konrad (2008) relate patent thickets to incentives for R&D investment. Also, Cockburn and MacGarvie (2009) focus on the initial acquisition of venture capital funding of start-up software companies facing patent thickets. Finally, Cockburn et al. (2010) look at innovative performance and licensing activities of firms impacted by patent thickets.

exposure, and expected stock returns. Moreover, by implementing tests that include firms in all industries, we show that the impact of patent thickets on firm value is a general pattern instead of an industry-specific phenomenon.[12] This study thus extends our understanding of patent ownership and has implications for stock investors.

The rest of the paper is organized as follows. In Section 2, we present a tractable model with the goal to deliver the key intuitions and theoretical predictions. In Section 3, we describe our data and present the empirical results for model implications and robustness checks. We conclude this paper with Section 4. In the Appendix, we provide all proofs as well as some ancillary explanations. Readers can refer to the Online Appendix for theoretical and empirical robustness.

2 Theoretical Discussion

In this session, we adopt a real option model to explain the effect of patent thickets on a firm's systematic risk exposure and expected stock returns. Specifically, we study a firm's behavior after a patent is granted, instead of its behavior in undertaking R&D projects to generate patents. A patent's value over time depends on the random evolution of market conditions and can be realized as a new product. Our model captures a firm's decision on the timing of such commercialization (also known as patent exploitation or option exercising).

2.1 Setup

We consider a single firm in the economy that holds a Lucas' tree (i.e., asset in place) producing an instantaneous dividend, θ_t, which follows a geometric Brownian motion:

$$\frac{d\theta_t}{\theta_t} = \mu dt + \sigma dz_t,$$

[12] Ziedonis (2004) and Cockburn et al. (2010) focus on the semiconductor industry and software industry respectively, while Cockburn and MacGarvie (2009) focus on start-up firms only.

where $\mu > 0$ and $\sigma > 0$. At $t = 0$, the firm receives an opportunity to plant a new tree (i.e., a new discovery or invention, hereafter a "patent" for short), which produces an instantaneous dividend $\xi\theta_t$ ($\xi > 0$) once it is successfully commercialized (i.e., transformed into an asset in place). We interpret a larger ξ as a higher initial value of the patent.

However, as the patent is surrounded by other patents granted to n owners, the patent will be successfully commercialized only if all n owners are compensated with royalty payments.[13] We interpret a larger n as a deeper patent thicket for model simplification.[14] We assume that at $t = 0$, the n owners negotiate with the firm simultaneously on royalty payments (i.e., owner i charges the firm a royalty payment, q_i, and pays a private cost, c_i, which can be the cost related to negotiations, litigations, or knowledge transfers when the patent is commercialized).[15] We define the patent exploitation cost to the firm as $Q \equiv \sum_{i=1}^{n} q_i$. Notably, our exploitation cost includes royalty payments and costs associated with patent litigations. Therefore, a firm with a higher Q is meant to pay higher royalty fees to and/or be under larger litigation threats from other patent owners.

The owners decide the royalty to be charged on the patent, and given these royalty payments, the firm decides the timing to exploit the patent. As a result, we can use backward induction to tackle the model. Given patent exploitation cost, Q, we first solve the real-option exercising problem to obtain the patent exploitation timing, τ^*, and the resulting price, P_t, for the firm. Next, with the firm's decision rule, we determine the royalty payments by solving a Cournot equilibrium among the owners of prior patents.

We assume that the stochastic discount factor, M_t, also follows a geometric Brownian

[13] Admittedly, patent commercialization costs may involve both royalty payments and litigation costs. In the Online Appendix, we analyze an extended model assuming that each owner of prior related patents will sue the focal firm with a certain probability when the firm exploits its patent.

[14] In this economy, the number of patent owners, n, captures the spirit of patent thickets. A firm with a larger n is confronted with a more fragmented market of innovation ownership, thereby suffering from a deeper patent thicket.

[15] In the basic model, we assume that the patent exploitation costs are determined in a simultaneous bargaining game, in which coordination among all patent owners is muted. To allow for coordination, we consider the sequential bargaining game in the Online Appendix.

motion:

$$\frac{dM_t}{M_t} = -rdt - \kappa dz_t,$$

where $r > 0$, $\kappa > 0$, and $\rho \equiv (r + \kappa\sigma - \mu)^{-1}$. To make the model meaningful, we also assume $r + \kappa\sigma - \mu > 0$. Note that, following Berk et al. (1999) and Zhang (2005), we parameterize the stochastic discount factor without explicitly modeling the consumer's problem. We argue that this modelling strategy is acceptable, as our focus is on firms' cash flows.

2.2 The Firm's Decision

The focal firm's price, P_t, includes two parts: the price of its asset in place, P_t^I, and the price of its patent, P_t^O. The price of asset in place is as follows,

$$P_t^I = E_t\left[\int_t^\infty \frac{M_s}{M_t}\theta_s ds\right] = \rho\theta_t, \tag{1}$$

and the price of patent is determined by the optimization problem of the stopping (patent exploitation) time, τ, which maximizes,

$$P_t^O = \sup_\tau E_t\left[\int_\tau^\infty \frac{M_s}{M_t}\xi\theta_s ds - \frac{M_\tau}{M_t}Q\right],$$

where the first term represents the present value of the (potential) real product, and the second term represents the present value of royalty paid. The following lemma characterizes the optimal stopping time (τ^*) and the price of patent (P_t^O).

Lemma 1 *The optimal stopping time $\tau^*$is reached when the market condition reaches θ^* (i.e., $\theta_{\tau^*} = \theta^*$), in which,*

$$\theta^* = \frac{\phi^+}{\phi^+ - 1}\frac{Q}{\rho\xi}, \text{ and} \tag{2}$$

$$\phi^+ = \frac{-\left(\mu - \kappa\sigma - \frac{1}{2}\sigma^2\right) + \sqrt{\left(\mu - \kappa\sigma - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2 r}}{\sigma^2} > 1.$$

The price of the patent is:

$$P_t^O = \left(\frac{\rho\xi}{\phi^+}\right)^{\phi^+}\left(\frac{\phi^+ - 1}{Q}\right)^{\phi^+ - 1}\theta_t^{\phi^+}. \tag{3}$$

See all proofs of lemmas and propositions in the Appendix.

From Equation (2), we have the following inequalities:

$$\frac{\partial \theta^*}{\partial Q} > 0, \text{ and} \tag{4}$$

$$\frac{\partial \theta^*}{\partial \xi} < 0. \tag{5}$$

From Equation (3), we find that,

$$\frac{\partial P_t^O}{\partial Q} < 0, \text{ and} \tag{6}$$

$$\frac{\partial P_t^O}{\partial \xi} > 0. \tag{7}$$

As a result, we obtain the following remarks.

Remark 1 *As the cost of patent exploitation (Q) increases, it is more difficult to transform a patent into an asset in place, so 1) the threshold of patent exploitation increases (Equation (4)), and 2) the price of the patent decreases (Equation (6)).*

Remark 2 *As the initial value of a patent (ξ) increases, it is more valuable to transform it into an asset in place earlier, so 1) the threshold of patent exploitation decreases (Equation (5)), and 2) the price of the patent increases (Equation (7)).*

Because the price of the firm is the sum of prices of its asset in place and patent, we obtain:

$$P_t = P_t^I + P_t^O. \tag{8}$$

Combining Equations (8) with (1) and (A4)[16] and applying Ito's lemma, we obtain:

$$dP_t = \left[\mu P_t^I + \phi^+ \mu P_t^O + \frac{1}{2}\phi^+ \left(\phi^+ - 1\right) \sigma^2 P_t^O\right] dt + \left[\sigma P_t^I + \phi^+ \sigma P_t^O\right] dz_t.$$

Therefore, the future stock return of the firm, R_t, can be defined as:

$$R_t \equiv \frac{dP_t + \theta_t dt}{P_t}$$

[16] See Appendix.

$$
\begin{aligned}
&= \frac{dP_t^I}{P_t^I + P_t^O} + \frac{dP_t^O}{P_t^I + P_t^O} + \frac{\theta_t}{P_t^I + P_t^O} dt \\
&= \mu dt + \frac{\Omega P_t^O + \theta_t}{P_t^I + P_t^O} dt + \frac{\sigma P_t^I + \phi^+ \sigma P_t^O}{P_t^I + P_t^O} dz_t,
\end{aligned} \tag{9}
$$

where $\Omega = (\phi^+ - 1)\,\mu + \phi^+\,(\phi^+ - 1)\,\sigma^2/2 > 0$, because $\phi^+ > 1$.

2.3 The Patent Owners' Decision

With the firm's optimal exploitation time, we proceed to derive the optimal royalty payments by solving a Cournot Equilibrium among the n owners. Owner i's maximization problem is given by:

$$
\max_{q_i} E_t \left[\frac{M_{\tau^*}}{M_t} (q_i - c_i) \right] = \max_{q_i} E_t \left[e^{-r(\tau^* - t)} (q_i - c_i) \right].
$$

Solving this optimization problem simultaneously for n owners, we have Lemma 2.

Lemma 2 *The optimal royalty payment, q_i^*, can be expressed as:*

$$
q_i^* = \frac{1}{\phi^+ - n} \left[\left(\phi^+ - n \right) c_i + \sum_{j=1}^{n} c_j \right]. \tag{10}
$$

When we assume all patent owners' private costs are the same ($c_i = c_j = c$), the optimum can be simplified to $q_i^* = q_j^* = q^*$ for $i, j = 1 \ldots n$, and

$$
q^* = \frac{\phi^+}{\phi^+ - n} c > c. \tag{11}
$$

For simplicity, we continue our discussion under the assumption that $c_i = c_j = c$.[17]

2.4 Theoretical Implications

From Equation (11) we can show that:

$$
\frac{dq^*}{dn} = \frac{\phi^+}{(\phi^+ - n)^2} c > 0, \text{ and} \tag{12}
$$

[17] In the Online Appendix, we relax this assumption and discuss the model implication under the case of $c_i \neq c_j$, where $i \neq j$.

$$\frac{dQ}{dn} = q^* + n\frac{dq^*}{dn} = \frac{(\phi^+)^2}{(\phi^+ - n)^2}c > 0. \tag{13}$$

Equation (13) shows the reasons why a deeper patent thicket leads to a higher exploitation cost: the population effect (the term q^*) as well as the coordination effect (the term $n \cdot dq^*/dn$). The population effect captures that given individual royalty fees, the total royalty cost increases with the number of patent owners surrounding the patent. Meanwhile, the coordination effect is in a similar spirit with Cournot and Fisher's (1897) "complements problem" and Heller and Eisenberg's (1998) "tragedy of anti-commons." Because of the complementary ownership relations, each owner will be better off if they can coordinate among themselves. However, each owner does not have any incentive to coordinate with others, as they share the public benefits of coordination but bear the private costs. Therefore, the positive externality of coordination induces them to over-exploit this "commons-like" patent, over-charge royalty payments, and hinder the exploitation of the patent.

We now illustrate the impact of a patent thicket from the perspective of asset pricing. A patent thicket, as we have shown, affects patent exploitation costs (Equation (13)) and exploitation timing (Equation (4)). Therefore, a patent thicket should influence the systematic risk exposure of the firm. Following the literature, the firm's total exposure to the systematic risk can be defined as:

$$\beta_t = -\frac{E_t\left[\frac{dM_t}{M_t}R_t\right]}{Var_t\left[\frac{dM_t}{M_t}\right]}.$$

Using the pre-specified property of the stochastic discount factor, M_t, and the return, dP_t/P_t, we can rewrite β_t as:

$$\beta_t = \frac{\sigma}{\kappa}\left[\frac{P_t^I + \phi^+ P_t^O}{P_t^I + P_t^O}\right]. \tag{14}$$

Taking the first derivative of β_t with respect to n, we have:

$$\frac{d\beta_t}{dn} = \frac{\sigma}{\kappa}\left[\frac{(\phi^+ - 1)\, P_t^I}{(P_t^I + P_t^O)^2}\right]\frac{dP_t^O}{dn}, \tag{15}$$

where the value in the bracket is always positive. Therefore, using Equations (6) and (13), we know:

$$\frac{d\beta_t}{dn} < 0,$$

which leads to Proposition 1.

Proposition 1 *A firm's exposure to systematic risk decreases with patent thickets.*

We further examine the relation between patent thickets and expected stock returns. From Equation (9), we take the first-order derivative of the expected stock return ($E_t[R_t]$) with respect to n and obtain:

$$\frac{dE_t[R_t]}{dn} = \left[\frac{\Omega P_t^I - \theta_t}{(P_t^I + P_t^O)^2}\right] \frac{\partial P_t^O}{\partial Q} \frac{dQ}{dn} dt.$$

In this equation, we impose the condition that $\theta_t < \Omega P_t^I$. When we assume the opposite, however, we obtain a counter-intuitive result: a firm with a deeper patent thicket has a higher expected stock return but a lower systematic risk exposure. Specifically, this condition ensures that the firm's expected stock return is dominated by the expected return yielded from the patent rather than by the dividend generated by the asset in place. In this case, as the partial derivative of P_t^O w.r.t. Q is negative (Equation (6)) and the derivative of Q with respect to n (Equation (13)) is positive, we know:

$$\frac{dE_t[R_t]}{dn} < 0, \tag{16}$$

which leads to Proposition 2.

Proposition 2 *A firm's expected stock return decreases with patent thickets.*

2.5 Summary and Possible Extensions

We build a real option model to examine the impact of patent thickets on a firm's systematic risk exposure and expected stock returns. The line of argument that underpins these two propositions is straightforward. A firm with a deeper patent thicket suffers from higher patent exploitation costs and delays its timing of exploiting the patent to produce real products. As it is more difficult for that firm to transform its patent into an asset in place, this firm is expected to receive lower stochastic future cash flows; therefore, this firm has a smaller exposure to systematic risk (Proposition 1). Because of its smaller exposure to systematic risk, this firm is expected to provide a lower future stock return (Proposition 2).

Our basic model is subject to limitations. First, while we assume that the focal firm has only one patent in the basic model, we derive an extended model to incorporate variable patent portfolio sizes in the Online Appendix. This extended model delivers not only the same model implications (namely, patent thickets decrease systematic risk exposure and expected stock returns), but also a new model implication: a firm's exposure to systematic risk and expected stock returns increase with patent portfolio sizes. This new model implication is supported by Tables V and VI in the paper and Table O.I in the Online Appendix. With respect to this new model implication, our intuition is straightforward: a firm with more patents experiences more growth opportunities and therefore is expected to receive more stochastic cash flows in the future.

Second, our basic model assumes that patent endowment is independent from patent thickets (i.e., a firm faces an exogenous patent thicket when it receives an endowment of patents). We adopt such a setting because 1) a firm does not know a patent thicket that it faces until its and competitors' patents are all granted, and also because 2) we focus on the cross-sectional variation of stock returns instead of time-series dynamics. We appreciate that some may question the extent to which our results will change if we consider that the initial value of patents (ξ) produced by a firm is subject to patent thickets the firm is facing. Moreover, one may realistically expect a firm will file a patent of higher value or quality when facing deeper patent thickets. We solve a model that takes this concern into account in our Online Appendix. In Section 3, we present further empirical discussions about the relation between patent thickets and patent quality.

In addition, we assume an exogenously given pricing kernel. One may understandably express concern that innovation, by its very nature, influences the pricing kernel. However, we argue that this concern should be minor, for when we focus on cross-sectional firm characteristics (i.e., patent thickets) instead of aggregate shocks, the cash flow effect should dominate the discount rate effect for a given firm. Meanwhile, the biggest advantage of our model is its tractability. With closed-form solutions, simple comparative statics can produce a rich set of implications.

The basic model assumes that the royalty fees are determined in a simultaneous bargaining game, in which coordination among all patent owners are muted. To allow some coordination, we consider the sequential bargaining game in our Online Appendix. Moreover, the basic model only considers royalty payment as the only cost caused by patent thickets. In another extended model in the Online Appendix, we consider litigation cost caused by patent thickets by letting each owner of prior patents sue the focal firm with a certain probability when the firm exploits its patent.

3 Empirical Analysis

In this section, we describe our data collection, empirical strategies, and test results for the propositions we developed in Section 2.

3.1 Data

Our sample includes firm-year observations in the intersection of stock transaction data from Center for Research in Security Prices (CRSP) with accounting data from Compustat. We then limit our sample to only firm-years with domestic common shares traded on NYSE, AMEX, and NASDAQ from July 1963 to June 2012, and exclude financial and other firms whose Fama-French 48 industry classification codes (Fama and French, 1997) are between

44 and 48 (banking, insurance, real estate, trading, and others).[18]

We then collect detailed information on patent litigations of public firms from 2000 to 2015 from Lex Machina, known as the most comprehensive database on patent litigations since 2000 (Akcigit et al., 2016).[19] We also collect new product announcements data from the News & Key Developments of Capital IQ databases, which record public firms's launching of new products since 2001.

Lastly, we collect these public firms' patent records from the patent database constructed by Kogan et al. (forthcoming) that includes detailed information (patent number, patent assignee's CRSP identifier, and patent grant date) on all U.S. patents granted by the U.S. Patent and Trademark Office (USPTO) between 1926 and 2010. However, the references (citations) made by every granted patent that is necessary for our patent thicket proxy is available only since 1962.

3.2 Empirical Measures

We discuss our empirical proxies for patent thickets, firm operations, and other control variables as follows.

Patent thicket. We use the definition of Ziedonis (2004) and define a firm's patent thicket index as an adjusted value of one minus the concentration index of the ownership of cited patents as follows:[20]

$$PT_{i,t} = Frag_{i,t} \cdot \frac{Numpats_{i,t}}{Numpats_{i,t} - 1}, \text{ when } Numpats_{i,t} > 1;$$

$$PT_{i,t} = Frag_{i,t}, \text{ when } Numpats_{i,t} = 1,$$

[18] We also exclude closed-end funds, trusts, American Depository Receipts, Real Estate Investment Trusts, units of beneficial interest, and firms with negative book equity, following Fama and French (1993).

[19] We thank Lex Machina for kindly providing us with their patent litigations data.

[20] This patent thicket measure has been used in many other studies, including Cockburn and MacGarvie (2009), Cockburn et al. (2010), Galasso and Schankerman (2010), Cockburn and MacGarvie (2011), Graevenitz et al. (2013), Noel and Schankerman (2013), and others.

where

$$Frag_{i,t} = 1 - \sum_{j=1}^{J} \left(\frac{Numcites_{i,t}^{j}}{Numcites_{i,t}} \right)^2, \; i \neq j.$$

$PT_{i,t}$ refers to an adjusted patent thicket index of firm i in year t, which is adjusted for downward bias following Hall (2005), and $Frag_{i,t}$ refers to an "ownership fragmentation index" of patents cited by firm i in year t, based on patents granted to firm i in years $t-4$ to year t.[21] $Numcites_{i,t}^{j}$ denotes the total number of citations made by firm i's patents granted in the most recent five years (from year $t-4$ to year t) to firm j's patents granted earlier ($j \neq i$),[22] and $Numcites_{i,t}$ stands for the total number of citations made by firm i's patents granted in the most recent five years (from year $t-4$ to year t) to all other firms' patents. By definition, $Numcites_{i,t} = \sum_{j=1, j\neq i}^{J} Numcites_{i,t}^{j}$. $Numpats_{i,t}$ represents the total number of firm i's patents granted in the most recent five years (from year $t-4$ to year t). When $Numpats_{i,t} > 1$, we use the adjustment factor $Numpats_{i,t}/(Numpats_{i,t}-1)$ to help mitigate the inflation driven by firm i's patent number, as firms owning more patents naturally make more citations.[23] Thus, our patent thicket proxy is patent portfolio size-neutral. Also, such size-neutrality facilitates our interpretation of test results, as they will not be subject to various size issues.

Based on the patent data of Kogan et al. (forthcoming), we calculate each public firm's annual PT index from 1981 to 2010 as it takes five years (1977-1981) to calculate the first PT value.[24] A high PT index suggests that a firm invents in a deeper patent thicket because it is

[21] It is common to use a five-year window to construct patent-based proxies related to innovation activities (Rothaermel and Deeds, 2004; Matolcsy and Wyatt, 2008; Hirshleifer et al., 2013).

[22] Citations to a firm's own patents are excluded from the proxy since they are unrelated to any patent royalty and infringement issues.

[23] Let us consider the following examples: (i) if firm i has three patents and each of them cites only one prior patent (owned by different firms), then its fragmentation index is $1 - 3 \times (0.3)^2 = 0.67$ before adjustment, and its PT is 1 after adjustment; (ii) if firm j has two patents and each of them cites only one prior patent (owned by different firms), its fragmentation index is $1 - 2 \times (0.2)^2 = 0.50$, and then its PT is also 1 after the adjustment; and (iii) if firm k has only one patent that cites only one prior patent, then its fragmentation index and PT are both 1. Since it is natural to cite more prior patents when a firm owns more patents, the adjustment helps us mitigate the inflation due to a firm's patent size.

[24] We start our PT measure in 1981 because the accounting treatment of R&D expense reporting was standardized in 1976, according to Financial Accounting Standards Board Statement No. 2, and we take one more year to ensure data comprehensiveness. Nevertheless, we find consistent results when we use the sample from 1962 to 2010 (not reported).

confronted with less concentrated (more fragmented) patent ownership. Based on our model, this firm will have to negotiate royalties or cross-licensing with more patent owners in order to exploit its growth option, such as commercializing a patent. With our model setting, we assume that such a negotiation process is costly and increases with the number of owners of related prior patents. Thus, alternative proxies of patent thickets would reflect the number of firms that are cited by a firm's patent portfolio (i.e., the number J). Although such a proxy is intuitive, it likely increases with the firm's patent portfolio size; in other words, a firm with more patents naturally cites prior patents from a wider range of firms and thus has a higher value of J.[25] Moreover, this proxy neglects the reality that a firm may directly exploit a patent without negotiating with some minor patent owners.[26] Such an action is reasonable because, even if the firm is sued for patent infringement later, the associated litigation risk and settlements associated with these minor shares will be lower.

Four important issues related to our patent thicket proxy merit further discussion. First, using the patent data of Kogan et al. (forthcoming), we identify patent ownership by their CRSP identifiers (permno), which require both firms i and j to be publicly listed.[27] As a robustness check, we construct an alternative patent thicket index using all patent assignees defined by PDPASS (including all entities such as universities, hospitals, governments, etc.) in the NBER Patent for patents granted in 1976-2006 (Hall et al., 2001) and obtain consistent results (unreported).

Second, we use the citations made by a firm's patents to measure its patent thicket, and we do so for two reasons. Different from European patent laws, U.S. patent laws require patent applicants to provide a complete list of references, including prior patents and documents

[25] In unreported tests, we replicate all of our analysis using the number J as the proxy for patent thickets and obtain qualitatively consistent results.

[26] Let us consider the following two scenarios: (i) if 50% of the citations made by firm i's only patent are to firm j and the other 50% of the citations made are to firm k, then firm i's PT is $1 - 2 \times (0.5)^2 = 0.5$; (ii) if 90% of the citations made by firm h's only patent are to firm j and the other 10% of the citations made are to firm k, then firm h's PT is $1 - (0.9)^2 - (0.1)^2 = 0.18$. In contrast to firm i, firm h will find it easier to exploit the patent without reaching an agreement with firm k because the expected penalty (if any) is relatively lower. Thus, our PT measure is better than J (which is 2 in both scenarios).

[27] In our sample, the proportion of patents granted to listed firms among all patents increases smoothly from about 35% to 40% from 1981 to 2010.

known to be relevant to the patentability of their applications ("duty of candor and good faith"). Such a reference list will later be reviewed and supplemented by patent examiners. If applicants failed to disclose all relevant prior patents, then their patent applications may be rejected by the USPTO (Caballero and Jaffe, 1993; Roach and Cohen, 2013) or their granted patents could be challenged or even invalidated by the court (Allison and Lemley, 1998; Sampat, 2010). More importantly, a firm is more likely to initiate an infringement litigation against other firms that cite its patents (Lanjouw and Schankerman, 2001). Thus, the complete reference list provided in a patent document allows us to track the owners of prior patents with whom a firm may have to negotiate in order to commercialize its patent.[28]

Third, we use the grant date of patents that is public information to construct our PT measure. Since granted patents and their references are fully disclosed by the United States Patent and Trademark Office (USPTO) in the weekly *Official Gazette*, the PT measure based on granted patents until year t is publicly observable at the end of year t (or the first week of year $t+1$) and can thus be used for portfolio sorting at the end of June of year $t+1$.

Fourth, our PT measure is silent with respect to the bargaining power of other patent owners because this measure only considers owners' shares of being cited rather than the strength of their patent portfolios. However, in reality, some patent owners are more powerful than others because they have stronger patent portfolios or are more aggressive in initiating patent lawsuits (Lanjouw and Schankerman, 2004). Therefore, we propose empirical proxies for the bargaining power of counterparties (i.e., the owners of patents cited by the focal firm) and control for them in robustness checks.

Patent litigation. We measure the litigation risk for a firm using the number of litigation cases against it (i.e., the focal firm being a defendant) over a five-year window.

Patent commercialization. We measure how effective a firm is able to commercialize its patents using the number of new products launched from year t to $t+4$ over the number

[28] Even if the list is incomplete or some citations it contains are irrelevant, it still reasonably approximates the fragmentation of patent ownership a firm faces unless the missing and irrelevant citations affect the distribution of patent ownership in any systematic way.

of patents granted from year $t-5$ to $t-1$.

Firm operations. Firm operations are measured across four dimensions: return on asset (ROA), return on equity (ROE), investment intensity (IA), and sales intensity (SA). ROA is defined as income before extraordinary items plus interest expenses divided by lagged total assets. ROE is measured by net income divided by lagged book equity. IA is defined as capital expenditure divided by lagged total assets. SA is defined as total sales divided by lagged total assets. Firm operational volatility is proxied by the standard deviation of one of these operation measures over a five-year window.

Innovation-related control variables. We consider patent portfolio size, bargaining power of counterparties, and R&D intensity. For patent portfolio size, we cannot directly use the total number of firm i's patents granted in the most recent five years (i.e., $Numpats_{i,t}$ mentioned above) as a proxy for patent portfolio size because this number often increases with the firm's asset in place. Thus, we measure the firm's patent portfolio size using $Numpats_{i,t}$ divided by the firm's book equity at the end of year t, denoted by CTBE. When a firm's CTBE is higher in a period, this firm is regarded as having a larger patent portfolio relative to its asset in place, and thus, the firm carries more growth options. Following Bessen (2004) and Noel and Schankerman (2013), we define the counterparties' bargaining power (denoted as BPC) of a firm as the total accumulative forward citations received by its counterparties in the previous five years until year t. Counterparties are defined as all other firms that have ever been cited by the focal firm in the previous five years. For R&D intensity, we follow Deng et al. (1999), Chan et al. (2001), and Lev et al. (2005) to use the accumulated R&D expenses in the previous five years (with a 20% obsolescence rate) scaled by book equity at the end of the current year. We denote R&D intensity as RDBE. According to Harhoff et al. (1999), Hall et al. (2005), and Moser et al. (2011), patent quality (denoted as PQ) is proxied with the total accumulative forward citations received by the patents that are granted to the focal firm in the previous five years.

Other control variables. We also consider market size and book-to-market ratio, two common firm characteristics used in the literature. We use a firm's stock market capitalization at the end of June of year $t+1$ as the proxy of market size (Size). A firm's book-to-market ratio (B/M) is defined as its book value of equity scaled by its market equity at the end of year t.

3.3 Summary Statistics

In Table I, we report the summary statistics of the PT measures for firms in each industry according to the Fama-French 48 (FF48) industry classification (Fama and French, 1997) since 1981. We first find that the industries with the most valid PT observations are those often regarded as high-tech, including medical equipment, pharmaceutical products chemicals, construction materials, machinery, business services, computers, and electronic equipment. The aircraft industry has the highest average PT (0.94), while the agriculture industry and the restaurants, hotels, and motels industry have the lowest average PT (0.72). In addition, the standard deviations of PT range from 0.07 to 0.36. These statistics suggest heterogeneity in industry-level patent thickets: some industries are highly fragmented in patent ownership, while some industries are less fragmented. To further relieve the concerns of bias potentially caused by industry heterogeneity, we control for industry fixed effects in regressions and form industry-balanced portfolios in portfolio sorting analyses.

[Table I here.]

In Table II Panel A, we present the time-series averages of cross-sectional averages of PT and other firm characteristics. We categorize all firm-year observations into six groups: for each year, firms with PT are assigned to quintile portfolios labelled "Low," "2," "3," "4," and "High" groups in order. Firms without PT are grouped together as the "No" group. On average, there are about 192 firms in each quintile and $3,536$ firms in the "No" group. Although there are many more firms without PT, the firms in the "No" group are much smaller. The average capitalizations (SIZE) of firms in the "No," "Low," "2," "3," "4," and

"High" groups are \$1.05, \$2.11, \$3.44, \$4.26, \$6.89, and \$6.22, respectively in trillions. In other words, firms with PT cover around 55% of the total stock market capitalization, which forms a substantial set that merits in-depth investigation from the perspectives of both stock market and innovation.

We also report average firm characteristics of quintile portfolios in Table II. We find more patent litigations and lower ratios of patent commercialization in higher PT portfolios. We do not find any clear pattern of patent portfolio size (CTBE) and bargaining power of counterparties (BPC) across quintile PT portfolios, which suggests that PT is a unique dimension of competition dynamics, as it is not associated with patent tournament and product race. Firms with PT are more R&D-intensive (measured by RDBE) than firms without PT. However, we also find that both R&D intensity and market beta slightly decrease with PT. Such a decreasing pattern can be interpreted as that R&D-intensive firms create more unique inventions and are thus less dependent on prior patents. The book-to-market ratios (B/M) of firms with a PT measure are much lower than those of firms without this measure. This finding makes sense, for firms with patents, a necessary condition for our PT calculation, often carry higher growth opportunities due to their innovativeness and patenting activities.

Of note, Table II Panel A suggests that firms with deeper patent thickets tend to create patents with higher quality that receive more subsequent citations (i.e., larger PQ). With our model, we argue that patent thickets affect a firm's patent exploitation behavior, because patent thickets potentially increase the costs (no matter in a form of royalty fees or litigations) of exploiting the firm's own patent ("usage-cost channel"). Admittedly, patent thickets may realistically affect stock returns by affecting a firm's patenting behaviors: expecting the usage right of a patent in a deeper thicket is more costly, a firm will patent an invention only if it is valuable enough. That is, patent thickets induce more cautious patenting behavior ("patenting-behavior channel"). As a result, while patent thickets lead to lower systematic risk exposure and expected stock returns through the usage-cost channel, these patent thickets may also lead to higher systematic risk exposure and expected stock returns through the patenting-behavior channel. In Sections 3.4.3, 3.4.4, and 3.4.5, we find that a deeper patent thicket is associated with lower systematic risk exposure, lower volatility in firm operations, and lower future stock returns. These results collectively confirm that the usage-cost channel denominates the patenting-behavior channel in data.

[Table II here.]

Table II Panel B presents the time-series averages of the cross-sectional Pearson correlation coefficients between PT and all other variables. We find that PT only weakly correlates with other firm characteristics. The Pearson correlation coefficients between PT and other competition-related variables (CTBE and BPC) are below 0.07 in absolute values. Such a weak correlation confirms our observation in Panel A that PT is a unique dimension of competition dynamics that are distinct from patent tournament and product race. On the other hand, although Panel A shows a decreasing pattern of R&D intensity along PT, the Pearson correlation coefficient between PT and RDBE is 0.00, which is low in absolute magnitude. In addition, PT is only weakly correlated with patent quality (the correlation is 0.06 with PQ).

PT positively correlates with patent litigation but negatively correlates with patent commercialization, which is consistent with Panel A and supports our model setting and implication that patent thickets increase exploitation costs for firms and thus delays their commercialization of patents. In addition, PT positively correlates with size and negatively correlates with the book-to-market ratio, consistent with Panel A.

3.4 Results

3.4.1 Future Patent Litigations and Patent Thickets

We argue that patent thickets increase coordination difficulty among previous patent owners and thereby increase exploitation costs of patents for new inventors, in a form of either licensing royalty or litigation. Although we are unaware of any comprehensive licensing royalty data source, we are able to collect the detailed information on patent litigations from Lex Machina. Using patent litigations data from 2000 to 2015, we run Poisson regressions to examine whether a firm's patent thicket in the current year can explain the actual number of litigation cases against this firm (i.e., the focal firm being a defendant) over the next five years. After merging the litigation records with firm-level data and ensuring that firm characteristics for which we control are non-missing, we end up with $11,490$ firm-year observations.

In the first regression specification (Column (1)) in Table III, we include patent thicket (PT) and control for only year fixed effects, and the coefficient of PT is positive and significant at the 1% level. For interpretative purposes, we use the ranked PT in Column (5), which takes the value of 1, 2, 3, 4, and 5 for firms with PT in the low, 2, 3, 4, and high quintiles, respectively in each year. Also, the coefficient of the ranked PT is positive and significant at the 1% level. In terms of economic magnitude, if a firm jumps from the lowest PT quintile to the highest, it will encounter 0.78 $(0.1954 \times (5-1))$ more litigation cases over the next five years. This amount is considerable, as the full-sample average number of five-year litigation cases is about 0.27. The monetary cost of this 0.78 more litigation cases is huge. According to the estimate by Bessen and Meurer (2012), when sued for patent infringement, alleged infringers lose about \$44.6 million of market value (in 2010 dollars). Benchmarked with this data, firms with the highest PT bear \$34.8 million more costs related to patent litigation.

When we include industry fixed effects to control for industry-specific litigation risk in Columns (2) and (6), the coefficients of PT and ranked PT are still positive and significant at the 1% level. Moreover, the economic magnitude is 28% $(4.9262/3.8596-1)$ or 40% $(0.2737/0.1954-1)$ higher. We also try to control for other firm characteristics in other columns, such as patent portfolio size over book equity (CTBE), counterparties' bargaining power (BPC), R&D capital over book equity (RDBE), market capitalization (Size), and book-to-market ratio (B/M). We find that the number of litigation cases over the next five years increases with counterparties' bargaining power, R&D intensity, and firm size, but decreases with patent portfolio size and book-to-market ratio. Controlling these variables does not weaken the explanatory power of PT for litigation. For example, controlling for year fixed effects, industry effects, and all control variables, a firm will encounter 0.73 ($0.1831 \times (5-1)$) more litigation cases over the next five years if that firm moves from the lowest PT quintile to the highest quintile (Column (8)). Overall, Table III supports a direct, positive linkage between patent thickets and future patent litigations, thereby verifying the underlying assumption of our basic model and confirming the extended model with litigation costs in the Online Appendix.

[Table III here.]

3.4.2 Future Patent Commercialization and Patent Thickets

The underlying mechanism through which patent thickets influence systematic risk exposure and expected stock return is that patent thickets increase exploitation costs of patents and therefore hampers the commercialization of these patents. Noting that launching new products is an important way for firms to commercialize their patents, our model predicts that a firm with a deeper patent thicket should delay its new products. To test this implication, we merge the new products data from 2001 to 2015 with firm-level patent thicket data and end up with $10,859$ firm-year observations. Then we regress a firm's ratio of patent commercialization over a five-year window that begins in the next year on its patent thicket in the current year. Our model predicts that the panel regression should generate a negative coefficient estimation of patent thickets.

The empirical results presented in Table IV strongly support the model prediction. In the regression specification where we include patent thicket (PT) and control for only year fixed effects (Columns (1) and (5)), the coefficients of PT are negative and statistically significant at the 1% level. The economic magnitude is also significant: if a firm jumps up by one rank from the lower PT quintile to the higher quintile, then it will launch 29% fewer new products out of its patent portfolio over the next five years.

When we control for year fixed effects, industry fixed effects, and all other control variables, such as patent portfolio size (CTBE), bargaining power of counterparties (BPC), R&D intensity (RDBE), market capitalization (Size), and book-to-market ratio (B/M), we still find significant explanatory power of PT for future patent commercialization. Also, the economic magnitude is significant: if a firm moves from the lowest PT quintile to the highest quintile, then its ratio of patent commercialization over the next five years will decrease by 44% ($-0.1104 \times (5-1)$). Generally speaking, Table IV shows a negative linkage between patent thickets and future patent commercialization, which supports our model implication that patent thickets delay the exploitation of patents.

[Table IV here.]

3.4.3 Future Market Betas and Patent Thickets

In the model, we propose that firms with deeper patent thickets have lower exposure to systematic risk (Proposition 1). To test this model implication, we proxy for a firm's systematic risk exposure by using its loading on the market factor. Specifically, for each year between 1981 and 2010 and for each firm, we regress its monthly excess stock returns on monthly excess market returns (MKT) using the future 60-month window and then estimate its time-series factor loading (market beta).[29] Next, we regress market betas estimated from the future 60 months (observations from year $t+1$ to $t+5$) on patent thickets in year t across all firms. We also try to control for year fixed effects, industry fixed effects, patent portfolio size (CTBE), bargaining power of counterparties (BPC), R&D intensity (RDBE), market capitalization (Size), and book-to-market ratio (B/M).

In Columns (1) and (5) of Table V where we control for only year fixed effects, the coefficient of PT is negative and significant at the 1% level. The economic magnitude is also

[29] For reliable estimation, we require any firm in our sample to have at least 48 months of observations in the 60-month window.

significant: a firm's market beta over the next five years will drop by 0.13 ($-0.0326 \times (5-1)$) if it moves from the lowest PT quintile to the highest quintile. This amount is non-negligible, for the average market beta is about 1.24 in the sample. Further controlling for industry fixed effects and other variables does not eliminate the explanatory power of PT on future market beta. Also, as shown in Column (8), the economic magnitude is significant: if a firm moves from the lowest PT quintile to the highest quintile, then its market beta over the next five years will plummet by 0.11 ($-0.0280 \times (5-1)$). In sum, Table V indicates that a firm's future market beta decreases with patent thickets and confirms Proposition 1.

Interestingly, we find in Columns (3), (4), (7), and (8) that higher patent portfolio size (CTBE) is associated with higher market beta over the next five years. This finding is consistent with the prediction of the extended model with variable patent portfolio size in the Online Appendix in that a firm's systematic risk exposure increases with its patent portfolio size.

[Table V here.]

3.4.4 Future Volatility in Firm Operation and Patent Thickets

With our model, we argue that firms with deeper patent thickets face more difficulties in transforming real options into assets in place; therefore, their risk exposure should be lower (Proposition 1) and also their volatility in future operations should be smaller. As Beaver et al. (1970) and Kothari et al. (2002) argue, earnings variability reflects a firm's systematic risk from an accounting perspective and is positively associated with market risk. We consider the following four dimensions of firm operations: ROA, ROE, investment intensity (IA), and sales-to-asset ratio (SA). Future operation volatility is proxied by the standard deviation of one of these variables over the next five years. When a firm commercializes its patents in the future, it should change its production and sales plans. As a result, firms with deeper patent thickets are expected to reveal less volatile profitability, investment, and sales in the future.

To examine the relation between patent thickets and future operations volatility, we regress future operations volatility on patent thickets in the current year. We also control for year fixed effects, industry fixed effects, patent portfolio size (CTBE), bargaining power of counterparties (BPC), R&D intensity (RDBE), market capitalization (Size), and book-to-market ratio (B/M). Table VI confirms the negative relation. Generally speaking, across various specifications, the coefficients of PT are negative and significant. For example, in Column (8), if a firm moves from the lowest PT quintile to the highest quintile, then its volatility in ROA over the next five years will fall by 0.02 ($-0.0053 \times (5-1)$, Panel A), volatility in ROE will plummet by 0.11 ($-0.0284 \times (5-1)$, Panel B), volatility in IA will drop by 0.01 ($-0.0016 \times (5-1)$, Panel C), and volatility in SA will slip by 0.02 ($-0.0053 \times (5-1)$, Panel D). These numbers are non-negligible compared with their sample means, as reported in Table II. In sum, Table VI supports the model implication that firms with deeper patent thickets experience few operational changes, due to delays in commercializing patents. Noteworthily, Table VI suggests that higher patent portfolio size (CTBE) is associated with higher volatility in firm operations over the next five years. Again, this empirical finding is consistent with the extended model we present in the Online Appendix that allows for variable patent portfolio size.

[Table VI here.]

3.4.5 Future Stock Returns and Patent Thickets

We use portfolio analyses to test Proposition 2 (i.e., firms with deeper patent thickets generate lower expected stock returns). At the end of June of year t from 1982 to 2011, we sort firms with non-missing PT measures into five PT groups (Low, 2, 3, 4, and High) based on the quintiles of PT in year $t-1$. In addition, we label firms with missing PT measures as members in the "No" group. We also construct a zero-cost portfolio by longing a unit of "High" PT portfolio and shorting a unit of "Low" PT portfolio and label this portfolio as "High-Low." Since our PT measure is based on granted patents and references made by these patents that are disclosed publicly in the USPTO's weekly *Official Gazette*, all information related to PT in year $t-1$ is publicly observable at the end of year $t-1$ (or the first week of year t). Nevertheless, to make our results comparable to prior studies, we use a six-month lag to form the PT portfolios at the end of June of year t and hold these portfolios for the next twelve months until June of year $t+1$. All portfolios are value-weighted, as we use each firm's lagged market capitalization to determine its weight in a portfolio.

Table VII reports the average monthly excess returns, and alphas and betas corresponding to the CAPM model of all five PT quintiles. Monthly excess return is monthly stock return in excess of the one-month Treasury bill rate. In the first column, we find a decreasing pattern in excess stock returns from the low group to the high group, as the average excess returns of the "Low," "2," "3," "4," and "High" groups are 0.95%, 0.90%, 0.77%, 0.71%, and 0.49% per month, respectively. The excess returns of the High-Low portfolio average −0.46% per month, which is statistically significant at the 1% level and provides preliminary support to Proposition 2. On the other hand, the average excess return of the No group is 0.63% per month, which is higher than those of the High PT group but lower than that of all other PT groups (Low to 4).

[Table VII here.]

As we have argued in the modeling part, expected stock returns decrease with patent thickets due to lower systematic risk exposure. To support this argument, we must examine the risk exposure of the PT portfolios on market risk. We implement time-series regressions by regressing excess portfolio returns on a single market factor in the CAPM model. The regression coefficients on the market factor are estimates of corresponding risk exposure for these portfolios.

We find supportive results to our model. First, we observe a decreasing pattern in market betas (as an imperfect measure of total systematic risk exposure), which confirms that firms with deeper patent thickets have lower expected returns because of their smaller exposure to the systematic risk. This relation is consistent with our earlier findings in Section 3.4.3.

In addition, the alphas estimated as the regression intercept terms decrease from the Low group to the High Group. The alpha of the High-Low portfolios for the CAPM model is −0.42%, which is statistically significant at the 1% level.

To further relieve the concerns of bias potentially caused by industry heterogeneity, we form industry-balanced PT portfolios. Specifically, we sort PT measures within each industry to ensure that all firms in each industry are evenly distributed across quintiles. Similar patterns show up in Table VIII. The average monthly industry-adjusted returns of the "Low," "2," "3," "4," and "High" groups are 0.99%, 0.90%, 0.67%, 0.75%, and 0.52%, respectively. The return of the High-Low portfolio averages −0.48%, which is statistically significant at the 1% level and further supports Proposition 2. Again consistent with our earlier findings in Section 3.4.3, the market beta of the High-Low portfolio is −0.08, statistically significant at the 1% level.

[Table VIII here.]

Our portfolio results presented in Tables VII and VIII also collectively indicate a unique role that patent thickets play in expected stock returns. On the one hand, both stock returns and market betas decrease with patent thickets, consistent with our model that such a relation can be attributed to lower systematic risk exposure of firms confronted with deeper patent thickets. However, on the other hand, we find that the "abnormal" returns adjusted for market risk (i.e., CAPM alphas) also decrease with patent thickets, and that the difference in alphas between the top quintile and the bottom quintile is statistically significant. This finding implies that empirically the CAPM model cannot fully explain the spreads in returns of portfolios sorted on patent thickets. Our interpretation is that the existing market factor does not perfectly capture the risk premia associated with technology dynamics.[30]

[30] Other explanations for significant alphas include misspecified regression models, non-linearity issues, and market frictions.

4 Concluding Remarks

In this paper, we examine the effect of fragmented patent ownership (patent thickets) in the financial market. We develop a model which proposes that the fragmented ownership of patents increases the costs of patent commercialization, reduces future stochastic cash flows, lowers systematic risk exposure, and thus leads to lower expected stock returns. These model implications are all supported by our empirical tests, for which we use various data sources. First of all, using patent litigations data, we confirm that firms with deeper patent thickets are more likely to engage in patent litigations as defendants over the next five years. Second, using new product announcements data, we find that firms surrounded by deeper patent thickets launch fewer new products relative to their patent portfolios over the next five years. Third, we find that these firms also have lower market betas and lower volatility in operations over the next five years. Finally, our portfolio analyses indicate that firms confronted with deeper patent thickets are associated with significantly lower stock returns over the next twelve months. We show that our results are robust to industry heterogeneity, patent market conditions, and well-documented firm characteristics. In sum, our theoretical model and empirical tests collectively point to an important role of patent thickets in equity pricing.

References

Aguerrevere, F. L., 2009. Real options, product market competition, and asset returns. Journal of Finance 64, 957–983.

Ai, H., Kiku, D., 2013. Growth to value: Option exercise and the cross section of equity returns. Journal of Financial Economics 107, 325–349.

Akcigit, U., Celik, M. A., Greenwood, J., 2016. Buy, keep, or sell: Economic growth and the market for ideas. Econometrica 84, 943–984.

Alcácer, J., Gittelman, M., Sampat, B., 2009. Applicant and examiner citations in US patents: An overview and analysis. Research Policy 38, 415–427.

Allison, J. R., Lemley, M. A., 1998. Empirical evidence on the validity of litigated patents. American Intellectual Property Law Association (AIPLA) Quarterly Journal 26, 185.

Baron, J., Pohlmann, T., 2015. The effect of patent pools on patenting and innovation-Evidence from contemporary technology standards. Working Paper .

Beaver, W., Kettler, P., Scholes, M., 1970. The association between market determined and accounting determined risk measures. Accounting Review 45, 654–682.

Berk, J. B., Green, R. C., Naik, V., 1999. Optimal investment, growth options, and security returns. Journal of Finance 54, 1553–1607.

Berk, J. B., Green, R. C., Naik, V., 2004. Valuation and return dynamics of new ventures. Review of Financial Studies 17, 1–35.

Bessen, J., 2004. Patent thickets: Strategic patenting of complex technologies. Working Paper .

Bessen, J., Meurer, M. J., 2012. The private costs of patent litigation. Journal of Law, Economics & Policy 9, 59–96.

Caballero, R. J., Jaffe, A. B., 1993. How high are the giants' shoulders: An empirical assessment of knowledge spillovers and creative destruction in a model of economic growth. In: *NBER Macroeconomics Annual 1993, Volume 8*, MIT press, pp. 15–86.

Carlson, M., Fisher, A., Giammarino, R., 2004. Corporate investment and asset price dynamics: Implications for the cross-section of returns. Journal of Finance 59, 2577–2603.

Chan, L. K., Lakonishok, J., Sougiannis, T., 2001. The stock market valuation of research and development expenditures. Journal of Finance 56, 2431–2456.

Clark, D. J., Konrad, K. A., 2008. Fragmented property rights and incentives for R&D. Management Science 54, 969–981.

Cockburn, I. M., MacGarvie, M. J., 2009. Patents, thickets and the financing of early-stage firms: Evidence from the software industry. Journal of Economics & Management Strategy 18, 729–773.

Cockburn, I. M., MacGarvie, M. J., 2011. Entry and patenting in the software industry. Management Science 57, 915–933.

Cockburn, I. M., MacGarvie, M. J., Mueller, E., 2010. Patent thickets, licensing and innovative performance. Industrial and Corporate Change 19, 899–925.

Cohen, L., Diether, K., Malloy, C., 2013. Misvaluing innovation. Review of Financial Studies 26, 635–666.

Cournot, A. A., Fisher, I., 1897. Researches into the mathematical principles of the theory of wealth. Macmillan Co.

Deng, Z., Lev, B., Narin, F., 1999. Science and technology as predictors of stock performance. Financial Analysts Journal 55, 20–32.

Fama, E. F., French, K. R., 1993. Common risk factors in the returns on stocks and bonds. Journal of Financial Economics 33, 3–56.

Fama, E. F., French, K. R., 1997. Industry costs of equity. Journal of Financial Economics 43, 153–193.

Galasso, A., Schankerman, M., 2010. Patent thickets, courts, and the market for innovation. RAND Journal of Economics 41, 472–503.

Garleanu, N., Panageas, S., Yu, J., 2012. Technological growth and asset pricing. Journal of Finance 67, 1265–1292.

Graevenitz, G., Wagner, S., Harhoff, D., 2013. Incidence and growth of patent thickets: The impact of technological opportunities and complexity. Journal of Industrial Economics 61, 521–563.

Greenwood, J., Hercowitz, Z., Krusell, P., 1997. Long-run implications of investment-specific technological change. American Economic Review 87, 342–362.

Hall, B., Jaffe, A., Trajtenberg, M., 2001. The NBER patent citations data file: Lessons, insights and methodological tools. Working Paper .

Hall, B. H., 2005. A note on the bias in herfindahl-type measures based on count data. Working Paper .

Hall, B. H., Jaffe, A., Trajtenberg, M., 2005. Market value and patent citations. RAND Journal of Economics pp. 16–38.

Harhoff, D., Narin, F., Scherer, F. M., Vopel, K., 1999. Citation frequency and the value of patented inventions. Review of Economics and Statistics 81, 511–515.

Heller, M. A., Eisenberg, R. S., 1998. Can patents deter innovation? The anticommons in biomedical research. Science 280, 698–701.

Hirshleifer, D., Hsu, P.-H., Li, D., 2013. Innovative efficiency and stock returns. Journal of Financial Economics 107, 632–654.

Hoberg, G., Phillips, G., 2012. The stock market, product uniqueness, and comovement of peer firms. Working Paper .

Hobijn, B., Jovanovic, B., 2001. The information-technology revolution and the stock market: Evidence. American Economic Review 91, 1203–1220.

Hou, K., Robinson, D. T., 2006. Industry concentration and average stock returns. Journal of Finance 61, 1927–1956.

Hsu, P.-H., 2009. Technological innovations and aggregate risk premiums. Journal of Financial Economics 94, 264–279.

Kogan, L., Papanikolaou, D., 2014. Growth opportunities, technology shocks, and asset prices. Journal of Finance 69, 675–718.

Kogan, L., Papanikolaou, D., Seru, A., Stoffman, N., forthcoming. Technological innovation, resource allocation, and growth. Quarterly Journal of Economics .

Kothari, S., Laguerre, T. E., Leone, A. J., 2002. Capitalization versus expensing: Evidence on the uncertainty of future earnings from capital expenditures versus R&D outlays. Review of accounting Studies 7, 355–382.

Laitner, J., Stolyarov, D., 2003. Technological change and the stock market. American Economic Review 93, 1240–1267.

Lanjouw, J. O., Schankerman, M., 2001. Characteristics of patent litigation: A window on competition. RAND Journal of Economics 32, 129–151.

Lanjouw, J. O., Schankerman, M., 2004. Protecting intellectual property rights: Are small firms handicapped? Journal of Law and Economics 47, 45–74.

Lev, B., Sarath, B., Sougiannis, T., 2005. R&D reporting biases and their consequences. Contemporary Accounting Research 22, 977–1026.

Lin, X., 2012. Endogenous technological progress and the cross-section of stock returns. Journal of Financial Economics 103, 411–427.

Matolcsy, Z. P., Wyatt, A., 2008. The association between technological conditions and the market value of equity. Accounting Review 83, 479–518.

Moser, P., Ohmstedt, J., Rhode, P., 2011. Patents, citations, and inventive output-evidence from hybrid corn. Working Paper .

Nelson, R. R. W., Sidney, G., 1982. An evolutionary theory of economic change. Harvard University Press, Cambridge, MA.

Noel, M., Schankerman, M., 2013. Strategic patenting and software innovation. Journal of Industrial Economics 61, 481–520.

Papanikolaou, D., 2011. Investment shocks and asset prices. Journal of Political Economy 119, 639–685.

Pástor, L., Veronesi, P., 2009. Technological revolutions and stock prices. American Economic Review 99, 1451–1483.

Roach, M., Cohen, W. M., 2013. Lens or prism? Patent citations as a measure of knowledge flows from public research. Management Science 59, 504–525.

Rothaermel, F. T., Deeds, D. L., 2004. Exploration and exploitation alliances in biotechnology: A system of new product development. Strategic Management Journal 25, 201–221.

Sampat, B. N., 2010. When do applicants search for prior art? Journal of Law and Economics 53, 399–416.

Shapiro, C., 1985. Patent licensing and R&D rivalry. American Economic Review pp. 25–30.

Shapiro, C., 2001. Navigating the patent thicket: Cross licenses, patent pools, and standard setting. In: *Innovation Policy and the Economy, Volume 1*, MIT press, pp. 119–150.

Shumpeter, J. A., 1934. The theory of economic development. Harvard University Press, Cambridge, MA.

Singh, J., Fleming, L., 2010. Lone inventors as sources of breakthroughs: Myth or reality? Management Science 56, 41–56.

Thompson, P., 2006. Patent citations and the geography of knowledge spillovers: Evidence from inventor- and examiner-added citations. Review of Economics and Statistics 88, 383–388.

Weitzman, M. L., 1998. Recombinant growth. Quarterly Journal of Economics 113, 331–360.

Zhang, L., 2005. The value premium. Journal of Finance 60, 67–103.

Ziedonis, R. H., 2004. Don't fence me in: Fragmented markets for technology and the patent acquisition strategies of firms. Management Science 50, 804–820.

A Appendix: Proofs

A.1 Proof of Lemma 1

Stochastic integration gives $E_t\left[\int_\tau^\infty M_s\theta_s/M_t ds\right] = E_t\left[\rho\theta_t e^{-r(\tau-t)}\right]$ and $E_t\left[\int_\tau^\infty M_\tau/M_t\right] = E_t\left[e^{-r(\tau-t)}\right]$, which simplifies the expression of the firm's expected profit from exploiting the patent to:

$$E_t\left[\int_\tau^\infty \frac{M_s}{M_t}\xi\theta_s ds - \frac{M_\tau}{M_t}Q\right] = E_t\left[e^{-r(\tau-t)}\left(\rho\xi\theta_t - Q\right)\right].$$

We construct a risk-free portfolio, H_t, by longing one unit of the patent, P_t^O, and shorting $\partial P_t^O/\partial\theta_t$ units of fundamental asset θ_t, and the process of this risk-free portfolio follows:

$$\begin{aligned} dH_t &= dP_t^O - \frac{\partial P_t^O}{\partial\theta_t}\left[d\theta_t + \left(r + \kappa\sigma - \mu\right)\theta_t dt\right] \\ &= \left(\frac{1}{2}\sigma^2\theta_t^2\frac{\partial^2 P_t^O}{\partial\theta_t^2} - \left(r + \kappa\sigma - \mu\right)\theta_t\frac{\partial P_t^O}{\partial\theta_t}\right)dt. \end{aligned}$$

Imposing $dH_t = rdt$ gives the following ODE of P_t^o in the continuation region:

$$\frac{1}{2}\sigma^2\theta_t^2\frac{\partial^2 P_t^O}{\partial\theta_t^2} + (\mu - \kappa\sigma)\theta_t\frac{\partial P_t^O}{\partial\theta_t} - rP_t^O = 0, \tag{A1}$$

which should satisfy three boundary conditions:

$$\begin{aligned} \text{Absorbing-Barrier Condition: } & P_t^O(\theta_t \to 0) < \infty; \\ \text{Value-Matching Condition: } & P_t^O(\theta_t = \theta^*) = \rho\xi\theta^* - Q; \\ \text{Smooth-Pasting Condition: } & \left.\frac{\partial P_t^O}{\partial\theta_t}\right|_{\theta_t=\theta^*} = \rho\xi. \end{aligned}$$

Equation (A1) yields the general solution:

$$P_t^O = D\theta_t^\phi. \tag{A2}$$

Plugging equation (A2) into (A1) leads to:

$$\phi^\pm = \frac{-\left(\mu - \kappa\sigma - \frac{1}{2}\sigma^2\right) \pm \sqrt{\left(\mu - \kappa\sigma - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2 r}}{\sigma^2}.$$

Because $r + \kappa\sigma - \mu > 0$, we have $\phi^+ > 1$ and $\phi^- < 0$. The Absorbing-Barrier Condition implies that ϕ^- should be dropped. Plugging $P_t^O = D\theta_t^{\phi^+}$ into the Value-Matching Condition and Smooth-Pasting Condition gives:

$$\theta^* = \frac{\phi^+}{\phi^+ - 1}\frac{Q}{\rho\xi}, \text{ and} \tag{A3}$$

$$P_t^O = \left(\frac{\rho\xi}{\phi^+}\right)^{\phi^+}\left(\frac{\phi^+ - 1}{Q}\right)^{\phi^+ - 1}\theta_t^{\phi^+}. \tag{A4}$$

A.2 Proof of Lemma 2

Because we have shown in the proof of Lemma 1 that:

$$P_t^O = \sup_{\tau} E_t\left[e^{-r(\tau - t)}\left(\rho\xi\theta_\tau - Q\right)\right],$$

we then obtain:

$$E_t\left[e^{-r(\tau^* - t)}\right] = \left[\frac{\phi^+ - 1}{\phi^+}\frac{\rho\xi}{Q}\right]^{\phi^+}\theta_t^{\phi^+}.$$

Therefore, the maximization problem of owner i is equivalent to:

$$\max_{q_i} \frac{q_i - c_i}{\left(\sum_{j=1}^n q_j\right)^{\phi^+}},$$

with the first-order condition (FOC) as:

$$\sum_{j=1}^n q_j - \phi^+(q_i - c) = 0. \tag{A5}$$

Summing the FOC across all i's, we have:

$$\sum_{j=1}^n q_j = \frac{\phi^+\sum_{j=1}^n c_j}{\phi^+ - n},$$

and, hence, the optimum can be obtained:

$$q_i^* = \frac{1}{\phi^+ - n}\left[\left(\phi^+ - n\right)c_i + \sum_{j=1}^n c_j\right].$$

If $c_i = c_j = c$, then $q_i = q_j = q^*$ for $j = 1...n$, and we have:

$$q^* = \frac{\phi^+}{\phi^+ - n} c.$$

Note that the expression leads to an implicit assumption that $\phi^+ > n$.

B Tables

Table I

Patent Thickets across Industries

This table reports the pooled mean (Mean), standard deviation (Std), minimum (Min), 25th percentile (Perc25), median (Perc50), 75th percentile (Perc75), and maximum (Max) of the patent thicket (PT) measure for firms in industries based on Fama-French 48 industry classifications (FF48), financial and other industries (44-48) excluded. The sample ranges from 1981 to 2010. Obs is the number of firm-year observations with non-missing PT measures in each industry.

FF48	Industry	Obs	Mean	Std	Min	Perc25	Perc50	Perc75	Max
1	Agriculture	36	0.72	0.34	0.00	0.73	0.83	0.93	1.00
2	Food Products	419	0.90	0.17	0.00	0.90	0.95	0.97	1.00
3	Candy & Soda	122	0.89	0.16	0.00	0.86	0.96	0.98	1.00
4	Beer & Liquor	75	0.93	0.07	0.57	0.91	0.96	0.98	1.00
5	Tobacco Products	68	0.91	0.08	0.67	0.87	0.96	0.97	1.00
6	Recreation	357	0.86	0.22	0.00	0.86	0.94	0.96	1.00
7	Entertainment	113	0.85	0.20	0.00	0.84	0.90	0.95	1.00
8	Printing and Publishing	102	0.77	0.34	0.00	0.80	0.93	0.96	1.00
9	Consumer Goods	922	0.86	0.21	0.00	0.86	0.93	0.96	1.00
10	Apparel	173	0.80	0.32	0.00	0.83	0.93	0.98	1.00
11	Healthcare	125	0.79	0.26	0.00	0.75	0.89	0.95	1.00
12	Medical Equipment	1,902	0.87	0.19	0.00	0.85	0.93	0.96	1.00
13	Pharmaceutical Products	2,390	0.82	0.25	0.00	0.80	0.91	0.95	1.00
14	Chemicals	1,162	0.90	0.16	0.00	0.91	0.95	0.97	1.00
15	Rubber and Plastic Products	341	0.86	0.21	0.00	0.85	0.93	0.97	1.00
16	Textiles	187	0.90	0.18	0.00	0.89	0.96	0.98	1.00
17	Construction Materials	1,018	0.89	0.21	0.00	0.90	0.96	0.98	1.00
18	Construction	84	0.81	0.31	0.00	0.83	0.94	0.97	1.00
19	Steel Works Etc	555	0.89	0.18	0.00	0.90	0.94	0.98	1.00
20	Fabricated Products	127	0.85	0.23	0.00	0.85	0.93	0.96	1.00
21	Machinery	2,312	0.91	0.16	0.00	0.92	0.95	0.97	1.00
22	Electrical Equipment	1,328	0.88	0.18	0.00	0.88	0.94	0.97	1.00
23	Automobiles and Trucks	846	0.91	0.16	0.00	0.91	0.95	0.97	1.00
24	Aircraft	331	0.94	0.11	0.00	0.95	0.97	0.98	1.00
25	Shipbuilding, Railroad Equipment	57	0.91	0.19	0.00	0.95	0.96	0.98	1.00
26	Defense	128	0.88	0.21	0.00	0.90	0.94	0.98	1.00
27	Precious Metals	13	0.85	0.27	0.00	0.81	0.94	1.00	1.00
28	Non-Metallic and Industrial Metal Mining	98	0.89	0.22	0.00	0.90	0.95	0.98	1.00
29	Coal	18	0.89	0.24	0.00	0.90	0.95	1.00	1.00
30	Petroleum and Natural Gas	679	0.90	0.18	0.00	0.92	0.95	0.97	1.00
31	Utilities	308	0.85	0.26	0.00	0.86	0.94	0.98	1.00
32	Communication	268	0.89	0.17	0.00	0.89	0.94	0.96	1.00
33	Personal Services	69	0.84	0.29	0.00	0.89	0.94	0.96	1.00
34	Business Services	2,346	0.86	0.21	0.00	0.87	0.92	0.95	1.00
35	Computers	1,758	0.90	0.13	0.00	0.90	0.94	0.96	1.00
36	Electronic Equipment	3,441	0.91	0.15	0.00	0.92	0.95	0.97	1.00
37	Measuring and Control Equipment	1,333	0.90	0.19	0.00	0.91	0.96	0.97	1.00
38	Business Supplies	611	0.91	0.15	0.00	0.92	0.95	0.97	1.00
39	Shipping Containers	286	0.92	0.12	0.00	0.90	0.95	0.97	1.00
40	Transportation	99	0.81	0.29	0.00	0.80	0.93	0.97	1.00
41	Wholesale	489	0.81	0.29	0.00	0.85	0.93	0.96	1.00
42	Retail	199	0.81	0.29	0.00	0.83	0.95	0.97	1.00
43	Restaraunts, Hotels, Motels	63	0.72	0.36	0.00	0.62	0.91	0.96	1.00

Table II

Summary Statistics by PT Groups and Correlation Matrix

At the end of June of year *t* from 1982 to 2011, we sort firms with non-missing patent thicket (PT) into five PT groups (Low, 2, 3, 4, and High) based on the quintiles of PT in year *t* – 1. In addition, we assign firms with missing PT (i.e., no patent) into the "No" group. The time-series average of cross-sectional mean characteristics of firms in each PT group is reported in Panel A, and the pooled correlation is reported in Panel B. the average number of observations denotes the average number of firms in each group across years. Patent portfolio size (CTBE) is the number of patents issued to a firm in the previous five years divided by the firm's book equity at the end of year *t* – 1. Bargaining power of counterparties (BPC, in thousands) is defined as the total accumulative forward citations received by the other firms which are cited by the focal firm until year *t* – 1. R&D intensity (RDBE) is accumulative R&D capital from year *t* – 1 to *t* – 5 (with a 20% obsolescence rate) in the fiscal year ending in year *t* – 1 divided by book equity at the end of year *t* – 1. Patent quality (PQ, in thousands) is the total accumulative forward citations of the patents in the previous five years of the focal firm. Litigation denotes a firm's number of patent litigation as defendant from year *t* – 1 to *t* – 5. The ratio of patent commercialization (PC) is defined as the number of new products launched from year *t* - 4 to *t* over the number of patents granted from year *t* - 9 to *t* - 5. Size is market capitalization (in trillions) at the end of June of year *t*. B/M is book-to-market ratio of the fiscal year ending in year *t* – 1. Beta denotes the estimated market beta from year *t* – 1 to *t* – 5. Std(ROA), Std(ROE), Std(IA), and Std(SA) denote the volatilities in ROA, ROE, IA, and SA in the previous five years, respectively. Return on assets (ROA) is defined as income before extraordinary items plus interest expenses in year *t* – 1 divided by total assets in year *t* – 2. Return on equity (ROE) is defined as net income in year *t* – 1 divided by total book equity in year *t* – 2. Investment intensity (IA) is defined as capital expenditure in year *t* – 1 divided by total assets in year *t* – 2. Sales-to-asset ratio (SA) is defined as total sales in year *t* – 1 divided by total assets in year *t* – 2.

Table II (continued)

Panel A						
PT group	**Low**	**2**	**3**	**4**	**High**	**No**
Average Number of Observations	194	192	193	192	192	3,536
Patent Thicket (PT)	0.61	0.90	0.94	0.96	0.99	----
Patent Portfolio Size (CTBE)	0.45	0.51	0.47	0.53	0.39	----
Bargaining Power of Counterparties (BPC)	17.29	21.46	22.38	21.92	18.27	----
R&D Intensity (RDBE)	0.25	0.24	0.23	0.19	0.22	0.09
Patent Quality (PQ)	0.63	2.04	4.16	6.42	4.26	----
Number of Patent Litigation (Litigation)	0.14	0.18	0.21	0.34	0.47	----
Patent Commercialization (PC)	0.45	0.29	0.30	0.22	0.30	----
Market Capitalization (Size)	2.11	3.44	4.26	6.89	6.22	1.05
Book-to-Market Ratio (B/M)	0.66	0.68	0.65	0.64	0.67	0.86
Market Beta (Beta)	1.18	1.28	1.31	1.28	1.14	0.95
Volatility in ROA [Std(ROA)]	0.10	0.09	0.09	0.07	0.06	0.07
Volatility in ROE [Std(ROE)]	0.44	0.34	0.30	0.28	0.20	0.41
Volatility in Investment [Std(IA)]	0.04	0.03	0.03	0.03	0.03	0.05
Volatility in Sales [Std(SA)]	0.27	0.25	0.24	0.23	0.22	0.30

Table II (continued)

Panel B	PT	CTBE	BPC	RDBE	PQ	Litigation	PC	Size	B/M	Beta	Std(ROA)	Std(ROE)	Std(IA)	Std(SA)
Patent Thicket (PT)	1	----	----	----	----	----	----	----	----	----	----	----	----	----
Patent Portfolio Size (CTBE)	0.01	1	----	----	----	----	----	----	----	----	----	----	----	----
Bargaining Power of Counterparties (BPC)	0.07	0.00	1	----	----	----	----	----	----	----	----	----	----	----
R&D Intensity (RDBE)	0.00	0.51	0.02	1	----	----	----	----	----	----	----	----	----	----
Patent Quality (PQ)	0.06	0.00	0.11	0.00	1	----	----	----	----	----	----	----	----	----
Number of Patent Litigation (Litigation)	0.02	0.01	0.14	0.02	0.10	1	----	----	----	----	----	----	----	----
Patent Commercialization (PC)	-0.03	-0.01	0.22	0.01	-0.02	0.05	1	----	----	----	----	----	----	----
Market Capitalization (Size)	0.06	-0.02	0.10	-0.01	0.37	0.24	0.01	1	----	----	----	----	----	----
Book-to-Market Ratio (B/M)	0.00	-0.04	-0.07	-0.04	-0.06	-0.03	-0.03	-0.10	1	----	----	----	----	----
Market Beta (Beta)	0.03	0.03	0.29	0.06	0.04	0.01	0.12	-0.09	-0.07	1	----	----	----	----
Volatility in ROA [Std(ROA)]	-0.05	0.05	0.08	0.06	-0.01	0.00	0.09	-0.04	-0.08	0.24	1	----	----	----
Volatility in ROE [Std(ROE)]	-0.04	0.03	0.03	0.03	-0.01	0.00	0.02	-0.02	-0.04	0.04	0.24	1	----	----
Volatility in Investment [Std(IA)]	-0.05	0.04	-0.05	0.02	-0.02	-0.03	-0.02	-0.06	-0.02	0.13	0.29	0.08	1	----
Volatility in Sales [Std(SA)]	-0.03	0.00	0.01	0.00	-0.02	-0.02	-0.01	-0.03	-0.02	0.14	0.20	0.06	0.47	1

Table III

Future Patent Litigations and Patent Thickets

We execute Poisson regressions to examine the relation between patent thickets and future patent litigations. In year *t* from 1999 to 2010, we run Poisson regressions of the accumulative number of future five-year litigation cases against the firm (from *t* + 1 to *t* + 5) on its patent thicket (PT) in the current period (computed from *t* to *t* – 4). We also control for year fixed effects, industry fixed effects, and CTBE, BPC (in millions), RDBE, Size (in millions), and B/M as described in Table II. Ranked PT takes the value of 1, 2, 3, 4, or 5 for firms with PT in the low, 2, 3, 4, and high quintiles, respectively in each year. Industry fixed effect (Industry FE) is constructed under Fama-French 48 industry classifications. We report the estimated coefficients and their standard errors (in parentheses). ***, **, and * indicate significance levels of 1%, 5%, and 10%, respectively.

Future Patent Litigation	Specification 1: Raw Patent Thicket				Specification 2: Ranked Patent Thicket			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Patent Thicket (PT)	3.8596***	4.9262***	3.6490***	3.4783***	0.1954***	0.2737***	0.1959***	0.1831***
	(0.1344)	(0.1520)	(0.1471)	(0.1446)	(0.0060)	(0.0067)	(0.0064)	(0.0072)
Patent Portfolio Size (CTBE)	----	----	-0.1522***	-0.1130***	----	----	-0.1284***	-0.1150***
	----	----	(0.0167)	(0.0188)	----	----	(0.0161)	(0.0185)
Bargaining Power	----	----	11.1941***	10.5685***	----	----	11.5596***	10.8182***
of Counterparties (BPC)	----	----	(0.2164)	(0.3276)	----	----	(0.2093)	(0.3167)
R&D Intensity (RDBE)	----	----	0.0956***	-0.1244***	----	----	0.0467***	-0.1107***
	----	----	(0.0104)	(0.0264)	----	----	(0.0079)	(0.0258)
Market Capitalization (Size)	----	----	0.0107***	0.0119***	----	----	0.0107***	0.0118***
	----	----	(0.0001)	(0.0001)	----	----	(0.0001)	(0.0001)
Book-to-Market Ratio (B/M)	----	----	-0.5115***	-0.2851***	----	----	-0.5376***	-0.2870***
	----	----	(0.0253)	(0.0232)	----	----	(0.0256)	(0.0234)
Constant	-4.3276***	-6.0047***	-4.5143***	-5.3289***	-1.4364***	-2.3955***	-1.8276***	-2.7567***
	(0.1263)	(0.1495)	(0.1400)	(0.1442)	(0.0292)	(0.0569)	(0.0350)	(0.0605)
Observations	11,490	11,490	11,490	11,490	11,490	11,490	11,490	11,490
Year FE	YES	YES	YES	YES	YES	YES	YES	YES
Industry FE	NO	YES	NO	YES	NO	YES	NO	YES

Table IV

Future Patent Commercialization and Patent Thickets

We run panel regressions to examine the relation between patent thickets and future new product commercialization. At each year *t* between 2000 and 2010, we regress the future new product commercialization (computed from year *t* + 1 to *t* + 5) on the current patent thicket (computed from year *t* – 4 to *t*). The ratio of patent commercialization is defined as the number of new products launched from year *t* + 1 to *t* + 5 over the number of patents granted from year *t* - 4 to *t*. We also control for year fixed effects, industry fixed effects, and CTBE, BPC (in millions), RDBE, Size (in trillions), and B/M described in Table II. Ranked PT takes the value of 1, 2, 3, 4, or 5 for firms with PT in the low, 2, 3, 4, and high quintiles, respectively in each year. Industry fixed effect (Industry FE) is constructed under Fama-French 48 industry classifications. Standard errors (in parentheses) are clustered by year and industry. ***, **, and * indicate significance levels of 1%, 5%, and 10%, respectively.

Future Patent Commercalization	Specification 1: Raw Patent Thicket				Specification 2: Ranked Patent Thicket			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Patent Thicket (PT)	-5.3040***	-4.9035***	-5.8315***	-4.9173***	-0.2864***	-0.1342**	-0.2912***	-0.1104*
	(0.5225)	(0.5290)	(0.5232)	(0.5310)	(0.0596)	(0.0616)	(0.0595)	(0.0620)
Patent Portfolio Size (CTBE)	----	----	-0.1977***	-0.1611***	----	----	-0.2119***	-0.1737***
	----	----	(0.0465)	(0.0461)	----	----	(0.0467)	(0.0463)
Bargaining Power	----	----	32.3188***	14.2882***	----	----	29.4086***	12.5525***
of Counterparties (BPC)	----	----	(2.8233)	(3.4313)	----	----	(2.8214)	(3.4432)
R&D Intensity (RDBE)	----	----	0.2865***	0.2133***	----	----	0.3081***	0.2326***
	----	----	(0.0740)	(0.0736)	----	----	(0.0743)	(0.0738)
Market Capitalization (Size)	----	----	-0.0066**	-0.0064**	----	----	-0.0071**	-0.0080**
	----	----	(0.0031)	(0.0032)	----	----	(0.0031)	(0.0032)
Book-to-Market Ratio (B/M)	----	----	-0.4441***	-0.3471***	----	----	-0.4314***	-0.3224**
	----	----	(0.1111)	(0.1272)	----	----	(0.1116)	(0.1277)
Constant	6.9437***	3.5911	7.0691***	11.0754*	3.1913***	-0.3886	2.9300***	6.5289
	(0.5279)	(4.9705)	(0.5323)	(5.7150)	(0.3236)	(4.9694)	(0.3374)	(5.7151)
Observations	10,859	10,859	10,859	10,859	10,859	10,859	10,859	10,859
R-squared	0.0170	0.0568	0.0318	0.0602	0.0097	0.0497	0.0229	0.0530
Year FE	YES	YES	YES	YES	YES	YES	YES	YES
Industry FE	NO	YES	NO	YES	NO	YES	NO	YES

Table V

Future Market Betas and Patent Thickets

We employ panel regressions to examine the relation between patent thickets and future market betas. In the first step, for each year between 1981 and 2010 and for each firm, we regress the firm's monthly excess return on the market factor (MKT) using the future 60-month window and get its time-series factor loading (market beta). A firm is included in our sample if it has at least 48 months of observation in the 60-month window. In the second step, we perform panel regressions of market betas estimated in the first stage (observations from year $t + 1$ to $t + 5$) on patent thickets in year t (computed from year $t - 4$ to t). We also try to control for year fixed effects, industry fixed effects, and CTBE, BPC (in millions), RDBE, Size (in trillions), and B/M, as described in Table II. Ranked PT takes the value of 1, 2, 3, 4, or 5 for firms with PT in the low, 2, 3, 4, and high quintiles, respectively in each year. Industry fixed effect (Industry FE) is constructed under Fama-French 48 industry classifications. Standard errors (in parentheses) are clustered by year and industry. ***, **, and * indicate significance levels of 1%, 5%, and 10%, respectively.

Future Market Beta	Specification 1: Raw Patent Thicket				Specification 2: Ranked Patent Thicket			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Patent Thicket (PT)	-0.1729***	-0.1739***	-0.2035***	-0.1942***	-0.0326***	-0.0321***	-0.0262***	-0.0280***
	(0.0415)	(0.0397)	(0.0429)	(0.0419)	(0.0045)	(0.0044)	(0.0044)	(0.0044)
Patent Portfolio Size (CTBE)	----	----	0.0143***	0.0071*	----	----	0.0142***	0.0071*
	----	----	(0.0042)	(0.0041)	----	----	(0.0042)	(0.0041)
Bargaining Power of Counterparties (BPC)	----	----	11.0962***	6.5837***	----	----	10.9550***	6.4249***
	----	----	(0.4331)	(0.4826)	----	----	(0.4327)	(0.4826)
R&D Intensity (RDBE)	----	----	0.0070***	0.0065***	----	----	0.0071***	0.0066***
	----	----	(0.0022)	(0.0022)	----	----	(0.0022)	(0.0022)
Market Capitalization (Size)	----	----	-0.3962***	-0.3222***	----	----	-0.3898***	-0.3133***
	----	----	(0.0247)	(0.0244)	----	----	(0.0248)	(0.0244)
Book-to-Market Ratio (B/M)	----	----	-0.0910***	-0.0556***	----	----	-0.0905***	-0.0566***
	----	----	(0.0134)	(0.0135)	----	----	(0.0134)	(0.0135)
Constant	1.2578***	0.7688***	1.3309***	0.8232***	1.1999***	0.7068***	1.2238***	0.7407***
	(0.0502)	(0.1605)	(0.0526)	(0.1612)	(0.0353)	(0.1578)	(0.0373)	(0.1583)
Observations	15,095	15,095	14,558	14,558	15,095	15,095	14,558	14,558
R-squared	0.0650	0.1762	0.1198	0.1937	0.0671	0.1781	0.1206	0.1948
Year FE	YES	YES	YES	YES	YES	YES	YES	YES
Industry FE	NO	YES	NO	YES	NO	YES	NO	YES

Table VI

Future Volatility in Firm Operations and Patent Thickets

We run panel regressions to examine the relation between patent thickets and future volatilities in firm operations. At each year *t* between 1981 and 2010, we regress the future volatility in firm operation (computed from year *t* + 1 to *t* + 5) on current patent thickets (computed from year *t* – 4 to *t*). We consider proxies of firm operations, such as profitability (ROA and ROE), investment ratio (IA), and sales ratio (SA). We also control for year fixed effects, industry fixed effects, and CTBE, BPC (in millions), RDBE, Size (in millions), and B/M, as described in Table II. Ranked PT takes the value of 1, 2, 3, 4, or 5, for firms with PT in the low, 2, 3, 4, and high quintiles, respectively in each year. Industry fixed effect (Industry FE) is constructed under Fama-French 48 industry classifications. Standard errors (in parentheses) are clustered by year and industry. ***, **, and * indicate significance levels of 1%, 5%, and 10%, respectively.

Panel A. Volatility in ROA

Future Volatility in Operation	Specification 1: Raw Patent Thicket				Specification 2: Ranked Patent Thicket			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Patent Thicket (PT)	-0.0332***	-0.0297***	-0.0351***	-0.0321***	-0.0069***	-0.0055***	-0.0064***	-0.0053***
	(0.0031)	(0.0031)	(0.0030)	(0.0029)	(0.0005)	(0.0005)	(0.0004)	(0.0004)
Patent Portfolio Size (CTBE)	----	----	0.0034***	0.0031***	----	----	0.0033***	0.0031***
	----	----	(0.0002)	(0.0002)	----	----	(0.0002)	(0.0002)
Bargaining Power of Counterparties (BPC)	----	----	0.2427***	0.1480**	----	----	0.1996***	0.1151*
	----	----	(0.0591)	(0.0642)	----	----	(0.0590)	(0.0642)
R&D Intensity (RDBE)	----	----	0.0024***	0.0022***	----	----	0.0024***	0.0022***
	----	----	(0.0003)	(0.0003)	----	----	(0.0003)	(0.0003)
Market Capitalization (Size)	----	----	-0.0005***	-0.0005***	----	----	-0.0005***	-0.0004***
	----	----	(0.0000)	(0.0000)	----	----	(0.0000)	(0.0000)
Book-to-Market Ratio (B/M)	----	----	-0.0082***	-0.0020*	----	----	-0.0079***	-0.0019*
	----	----	(0.0010)	(0.0010)	----	----	(0.0010)	(0.0010)
Constant	0.0249	0.0542	0.0554***	0.0486***	0.0142	0.0446	0.0434***	0.0386**
	(0.0621)	(0.0630)	(0.0101)	(0.0180)	(0.0620)	(0.0629)	(0.0098)	(0.0178)
Observations	26,462	26,462	25,051	25,051	26,462	26,462	25,051	25,051
R-squared	0.0722	0.1215	0.1054	0.1516	0.0756	0.1230	0.1080	0.1525
Year FE	YES	YES	YES	YES	YES	YES	YES	YES
Industry FE	NO	YES	NO	YES	NO	YES	NO	YES

Table VI (continued)

Panel B. Volatility in ROE								
Future Volatility in Operation	**Specification 1: Raw Patent Thicket**				**Specification 2: Ranked Patent Thicket**			
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
Patent Thicket (PT)	-0.1194*	-0.1214*	-0.1207*	-0.1343*	-0.0327***	-0.0288***	-0.0290***	-0.0284***
	(0.0667)	(0.0675)	(0.0685)	(0.0694)	(0.0101)	(0.0103)	(0.0101)	(0.0103)
Patent Portfolio Size (CTBE)	----	----	0.1312***	0.1317***	----	----	0.1311***	0.1316***
	----	----	(0.0044)	(0.0044)	----	----	(0.0044)	(0.0044)
Bargaining Power of Counterparties (BPC)	----	----	-1.9845	-0.8235	----	----	-2.1544	-0.9798
	----	----	(1.3644)	(1.5176)	----	----	(1.3637)	(1.5172)
R&D Intensity (RDBE)	----	----	0.1814***	0.1811***	----	----	0.1815***	0.1812***
	----	----	(0.0068)	(0.0068)	----	----	(0.0068)	(0.0068)
Market Capitalization (Size)	----	----	-0.0016*	-0.0018**	----	----	-0.0015*	-0.0017**
	----	----	(0.0008)	(0.0008)	----	----	(0.0008)	(0.0008)
Book-to-Market Ratio (B/M)	----	----	-0.0481**	-0.0529**	----	----	-0.0473**	-0.0533**
	----	----	(0.0238)	(0.0248)	----	----	(0.0238)	(0.0248)
Constant	0.0912	0.1600	0.1525	0.2055	0.0662	0.1310	0.1066	0.1525
	(1.3302)	(1.3823)	(1.5674)	(1.6083)	(1.3294)	(1.3817)	(1.5662)	(1.6073)
Observations	26,451	26,451	25,041	25,041	26,451	26,451	25,041	25,041
R-squared	0.0030	0.0085	0.0854	0.0908	0.0033	0.0087	0.0856	0.0910
Year FE	YES	YES	YES	YES	YES	YES	YES	YES
Industry FE	NO	YES	NO	YES	NO	YES	NO	YES

Table VI (continued)

Panel C. Volatility in IA								
Future Volatility in Operation	Specification 1: Raw Patent Thicket				Specification 2: Ranked Patent Thicket			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Patent Thicket (PT)	-0.0081***	-0.0086***	-0.0088***	-0.0091***	-0.0016***	-0.0015***	-0.0016***	-0.0016***
	(0.0011)	(0.0011)	(0.0010)	(0.0010)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
Patent Portfolio Size (CTBE)	----	----	0.0004***	0.0004***	----	----	0.0004***	0.0004***
	----	----	(0.0001)	(0.0001)	----	----	(0.0001)	(0.0001)
Bargaining Power of Counterparties (BPC)	----	----	0.0559***	-0.0499**	----	----	0.0449**	-0.0595***
	----	----	(0.0205)	(0.0225)	----	----	(0.0205)	(0.0224)
R&D Intensity (RDBE)	----	----	0.0000	0.0000	----	----	0.0000	0.0000
	----	----	(0.0001)	(0.0001)	----	----	(0.0001)	(0.0001)
Market Capitalization (Size)	----	----	-0.0001***	-0.0001***	----	----	-0.0001***	-0.0001***
	----	----	(0.0000)	(0.0000)	----	----	(0.0000)	(0.0000)
Book-to-Market Ratio (B/M)	----	----	-0.0046***	-0.0042***	----	----	-0.0045***	-0.0042***
	----	----	(0.0004)	(0.0004)	----	----	(0.0004)	(0.0004)
Constant	0.0436***	0.0471***	0.0466***	0.0520***	0.0415***	0.0448***	0.0438***	0.0494***
	(0.0028)	(0.0064)	(0.0027)	(0.0059)	(0.0027)	(0.0064)	(0.0026)	(0.0058)
Observations	26,219	26,219	24,822	24,822	26,219	26,219	24,822	24,822
R-squared	0.0246	0.0604	0.0366	0.0737	0.0261	0.0612	0.0383	0.0749
Year FE	YES	YES	YES	YES	YES	YES	YES	YES
Industry FE	NO	YES	NO	YES	NO	YES	NO	YES

Table VI (continued)

Panel D. Volatility in SA								
Future Volatility in Operation	**Specification 1: Raw Patent Thicket**				**Specification 2: Ranked Patent Thicket**			
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
Patent Thicket (PT)	-0.0511***	-0.0527***	-0.0563***	-0.0570***	-0.0047***	-0.0047***	-0.0051***	-0.0053***
	(0.0070)	(0.0070)	(0.0064)	(0.0064)	(0.0011)	(0.0011)	(0.0009)	(0.0010)
Patent Portfolio Size (CTBE)	----	----	0.0058***	0.0055***	----	----	0.0058***	0.0055***
	----	----	(0.0004)	(0.0004)	----	----	(0.0004)	(0.0004)
Bargaining Power of Counterparties (BPC)	----	----	1.1248***	0.2932**	----	----	1.0709***	0.2466*
	----	----	(0.1280)	(0.1407)	----	----	(0.1281)	(0.1408)
R&D Intensity (RDBE)	----	----	0.0002	0.0003	----	----	0.0003	0.0004
	----	----	(0.0006)	(0.0006)	----	----	(0.0006)	(0.0006)
Market Capitalization (Size)	----	----	-0.0007***	-0.0006***	----	----	-0.0007***	-0.0006***
	----	----	(0.0001)	(0.0001)	----	----	(0.0001)	(0.0001)
Book-to-Market Ratio (B/M)	----	----	0.0080***	0.0064***	----	----	0.0085***	0.0068***
	----	----	(0.0022)	(0.0023)	----	----	(0.0022)	(0.0023)
Constant	0.0437	0.2398*	0.2829***	0.3778***	0.0175	0.2146	0.2489***	0.3480***
	(0.1404)	(0.1442)	(0.0219)	(0.0393)	(0.1405)	(0.1442)	(0.0214)	(0.0391)
Observations	26,462	26,462	25,051	25,051	26,462	26,462	25,051	25,051
R-squared	0.0171	0.0459	0.0326	0.0613	0.0158	0.0446	0.0307	0.0595
Year FE	YES	YES	YES	YES	YES	YES	YES	YES
Industry FE	NO	YES	NO	YES	NO	YES	NO	YES

Table VII
One-way Portfolio Sorting on Patent Thickets

At the end of June of year *t* from 1982 to 2011, we sort firms with non-missing patent thicket (PT) into five PT groups (Low, 2, 3, 4, and High) based on the quintiles of PT in year *t* – 1. In addition, we assign firms with missing PT (i.e., no patent) into the "No" group. We also construct a high-minus-low (High–Low) portfolio by holding a long (short) position in the high (low) PT portfolio and hold these portfolios over the next twelve months (July of year *t* to June of year *t* + 1). We report the monthly returns in excess of the one-month Treasury bill rate (Excess Returns) and their corresponding alphas and betas in the CAPM model. MKT denotes the market factor. All returns and alphas are value-weighted and in percentage. The numbers in parentheses denote the standard errors. ***, **, and * indicate significance levels of 1%, 5%, and 10%, respectively. Sample period: July 1982 to June 2012.

PT	Excess Return	CAPM	
	Time-series Mean	Alpha	MKT
No	0.63**	-0.01	0.99***
	(0.25)	(0.06)	(0.01)
Low	0.95***	0.32**	0.98***
	(0.27)	(0.13)	(0.03)
2	0.90***	0.19	1.10***
	(0.31)	(0.16)	(0.04)
3	0.77***	0.10	1.03***
	(0.27)	(0.11)	(0.02)
4	0.71***	0.10	0.95***
	(0.25)	(0.08)	(0.02)
High	0.49**	-0.10	0.91***
	(0.24)	(0.09)	(0.02)
High-Low	-0.46***	-0.42***	-0.07**
	(0.16)	(0.16)	(0.03)

Table VIII

One-way Portfolio Sorting on Patent Thickets with Industry Adjustments

We rule out industry heterogeneity by doing portfolio sorting within industries. At the end of June of year *t* from 1982 to 2011, we sort firms with non-missing patent thicket (PT) into five PT groups (Low, 2, 3, 4, and High) based on the quintiles of PT in year *t* – 1 and in each industry. We also construct a high-minus-low (High–Low) portfolio by holding a long (short) position in the high (low) PT portfolio and hold these portfolios over the next twelve months (July of year *t* to June of year *t* + 1). We report the monthly returns in excess of the one-month Treasury bill rate (Excess Returns) and their corresponding alphas and betas in the CAPM model. MKT denotes the market factor. The numbers in parentheses denote the standard errors. ***, **, and * indicate significance levels of 1%, 5%, and 10%, respectively. Sample period: July 1982 to June 2012.

PT	Excess Return	CAPM	
	Time-series Mean	Alpha	MKT
Low	0.99***	0.36***	0.98***
	(0.26)	(0.10)	(0.02)
2	0.90***	0.19	1.10***
	(0.30)	(0.13)	(0.03)
3	0.67**	-0.01	1.06***
	(0.28)	(0.12)	(0.03)
4	0.75***	0.16*	0.92***
	(0.24)	(0.08)	(0.02)
High	0.52**	-0.06	0.90***
	(0.24)	(0.09)	(0.02)
High-Low	-0.48***	-0.43***	-0.08***
	(0.14)	(0.14)	(0.03)

1 Online Appendix: Model Extensions

1.1 Endogeneity of Patent Endowment

In our model, we assume patent value (ξ) and patent thicket (n) are independent. However, in reality, patent value may be negatively related with patent thickets. We argue that a firm ex ante will only file a patent of sufficiently high value if it expects the costs associated with patent thickets will offset its profit ex post. In the following proposition, we show that the effect of patent thickets on systematic risk exposure and expected stock returns when patent value is negatively related with patent thickets is weaker than the effect when patent value is unrelated with patent thickets.

Proposition O1 *The negative relation between patent value and patent thicket weakens the negative effect of patent thicket on systematic risk exposure and expected stock return.*

Proof. *Under the assumption that ξ is a continuous and differentiable function of n, such that $d\xi/dn > 0$, Equation (15) can be written as:*

$$\frac{d\beta_t}{dn} = \frac{\sigma}{\kappa}\left[\frac{(\phi^+ - 1)\, P_t^I}{(P_t^I + P_t^O)^2}\right]\left[\frac{\partial P_t^O}{\partial Q}\frac{dQ}{dn} + \frac{\partial P_t^O}{\partial \xi}\frac{d\xi}{dn}\right].$$

Because we know $\partial P_t^O/\partial Q < 0$, $dQ/dn > 0$, and $\partial P_t^O/\partial \xi > 0$, we have:

$$\frac{d\beta_t}{dn}|_{d\xi/dn=0} < \frac{d\beta_t}{dn}|_{d\xi/dn>0}.$$

Also from Equation (9), we can prove the following inequality:

$$\frac{dE_t\left[R_t\right]}{dn}|_{d\xi/dn=0} < \frac{dE_t\left[R_t\right]}{dn}|_{d\xi/dn>0}.$$

■

1.2 Effect of Litigation Costs

In the paper, we assume that the focal firm pays explicit royalties to the previous patent owners when it commercializes its patent. We consider another form of patent commercialization costs, which is litigation cost.

We assume that the focal firm will not pay royalties to the previous patent owners, but it faces risks of litigation. When it commercializes its patent, each of the patent owners will sue it with probability of p. Being involved in a litigation case is costly to the focal firm, the cost being c. Both p and c are given positive parameters.

Under these new assumptions, the patent exploitation cost to the focal firm is:

$$Q_L = npc.$$

The above equation implies that the patent exploitation cost is increasing with patent thicket, i.e.,

$$\frac{dQ_L}{dn} > 0. \tag{O1}$$

Equation (13) in the paper and Equation (O1) in this online-appendix section show that both models with royalty payments and litigation costs lead to the same set of implications (i.e., Proposition 1 and Proposition 2).

1.3 Effect of Patent Owners' Bargaining Power

The effect of patent owners' individual bargaining power on innovation competition and technology exploitation could be different. For example, Lanjouw and Schankerman (2004) show that small patent owners suffer from higher cost when protecting their patents. In the following two subsections, we take these two effects into consideration.

1.3.1 Effect of Heterogeneous Private Costs

In this subsection, we assume patent owners have different private cost c_i. We can prove the following proposition.

Proposition O2 *The patent owners suffering from higher private cost c_i are more influential to the systematic risk exposure and the expected stock return of the focal firm.*

Proof. *Equation (10) can by written as:*

$$q_i^* = \frac{1}{\phi^+ - n}\left[\left(\phi^+ - n + 1\right) c_i + \sum_{j=1, j\neq i}^{n} c_j\right].$$

So we have,

$$\frac{\partial q_i^*}{\partial c_i} > \frac{\partial q_i^*}{\partial c_j} > 0. \tag{O2}$$

Combining Inequalities (O2) and (14) yields:

$$\frac{\partial \beta_t}{\partial c_i} < 0.$$

Alsoo, combining Inequalities (O2) and (9) yields:

$$\frac{\partial E_t\left[R_t\right]}{\partial c_i} < 0.$$

■

1.3.2 Effect of Different Bargaining Timing

A sequential (Stackelberg) bargaining game is considered in this subsection, where patent owner i moves first to charge a royalty fee q_i, and the rest $(n-1)$ of them move second to charge q_{-i}. We assume that all owners have the same level of private costs c, in order to rule out the effect of private costs. We have the following proposition.

Proposition O3 *The patent owners, who move first to initiate patent negotiation, charge a higher royalty fee from the focal firm, and are more influential to the systematic risk exposure and the expected stock return of the focal firm.*

Proof. *For the second stage of the Stackelberg game, the first-order condition of the optimization problem of patent owners can be reorganized as:*

$$\sum_{-i} q_{-i} + q_i + \phi^+ (q_{-i} - c) = 0.$$

Summing it across the $n-1$ *patent owners yields:*

$$(n-1)\sum_{-i} q_{-i} + (n-1)\, q_i + \phi^+ \left(\sum_{-i} q_{-i} - (n-1)\, c \right) = 0,$$

or its equivalent:

$$q_{-i} = \frac{\phi^+ c - q_i}{\phi^+ + n - 1}. \tag{O3}$$

For the first stage of the Stackelberg game, the profit maximization problem is:

$$\max_{q_i} \frac{q_i - c}{\left(\sum_{-i} q_{-i}(q_i) + q_i\right)^{\phi^+}},$$

with the FOC:

$$\left(\sum_{-i} q_{-i}(q_i) + q_i \right) - \phi^+ \left((n-1)\frac{dq_{-i}}{dq_i} + 1 \right)(q_i - c) = 0.$$

Given Equation (O3), the FOC above can be solved as:

$$q_i^* = \frac{\phi^+ + n - 1}{\phi^+ - 1} c < \frac{\phi^+}{\phi^+ - n} c, \tag{O4}$$

and plugging Equantion (O4) into Equation (O3) gives:

$$q_{-i}^* = \frac{(\phi^+ - 1)^2 - n}{(\phi^+ + n - 1)(\phi^+ - 1)} c < \frac{\phi^+}{\phi^+ - n} c. \tag{O5}$$

Comparing Equation (O4) with Equation (O5) yields:

$$q_i^* > q_{-i}^*. \tag{O6}$$

Combining Inequalities (O6) and (14) yields:

$$\frac{\partial \beta_t}{\partial q_i^*} < \frac{\partial \beta_t}{\partial q_{-i}^*}.$$

Also, combining Inequalities (O6) and (9) yields:

$$\frac{\partial E_t [R_t]}{\partial q_i^*} < \frac{\partial E_t [R_t]}{\partial q_{-i}^*}.$$

■

1.4 An Extended Model with Variable Patent Portfolio Size

In this section, we extend the model in the paper to incorporate variable patent portfolio size (i.e., the number of patents a firm holds). This new feature enables us to further understand the impact of patent thickets on asset prices and generalize new insights.

This economy has a single firm and n owners surrounding each patent granted to the firm. The property of asset in place, stochastic discount factor, and patents are the same as described in the main model. We add patent portfolio size into the model by assuming that at $t = 0$ the firm is endowed with A identical and independent patents.[1] Hereafter, we interpret a large value of A as a bigger patent portfolio. In this model, the firm decides the timing of patent exploitation given the royalty fees, whereas the owners of surrounding patents decide the royalty fees to collect from the firm when the focal firm exploits the patent. As a result, it leads to a similar backward induction procedure as in the main model.

1.4.1 The Firm's Decision

The focal firm's price P_t includes the price of its asset in place P_t^I and the price of patents AP_t^O. Similar to the Basic Model, its price of asset in place is given by $P_t^I = \rho\theta_t$ in Equation (1). However, the price of patent is modified to the optimization problem of the stopping time (τ) that maximizes:

$$P_t^O = \sup_{\tau} E_t \left[\int_{\tau}^{\infty} \frac{M_s}{M_t} \xi \theta_s ds - \frac{M_\tau}{M_t} Q \right],$$

[1] We assume that these A patents are identical but independent because we would like to show the "mean" effect of patent thickets on asset prices. We could show that when these A patents are substitutes, the effect of patent thickets is relieved to some extent, and that when these A patents are complements, the effect of patent thickets is even more severe.

in which the first term represents the present value of the (potential) real product, and the second term represents the present value of royalty paid.

The following lemma characterizes the optimal stopping time and the price of patent.

Lemma O1 *The optimal stopping time is reached when the market condition reaches θ^* (i.e., $\theta_{\tau^*} = \theta^*$), in which:*

$$\theta^* = \frac{\phi^+}{\phi^+ - 1} \frac{Q}{\rho\xi}, \text{ and} \tag{O7}$$

$$\phi^+ = \frac{-\left(\mu - \kappa\sigma - \frac{1}{2}\sigma^2\right) + \sqrt{\left(\mu - \kappa\sigma - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2 r}}{\sigma^2} > 1.$$

The price of patent is:

$$P_t^O = \left(\frac{\rho\xi + b\rho}{\phi^+}\right)^{\phi^+} \left(\frac{\phi^+ - 1}{Q - a/r}\right)^{\phi^+ - 1} \theta_t^{\phi^+}. \tag{O8}$$

As we did in the main model, we use Equations (O7) and (O8) to obtain the following inequalities: $\partial\theta^*/\partial Q > 0$, $\partial\theta^*/\partial\xi < 0$, $\partial P_t^O/\partial Q < 0$, and $\partial P_t^O/\partial\xi > 0$. As a result, Remark 1 and Remark 2 are valid in the full model as well.

Because the price of the firm is the sum of its asset in place and patents, we obtain:

$$P_t = P_t^I + AP_t^O. \tag{O9}$$

Applying Ito's lemma to Equations (O9), we have:

$$dP_t = \left[\mu P_t^I + \phi^+ \mu A P_t^O + \frac{1}{2}\phi^+ \left(\phi^+ - 1\right) \sigma^2 A P_t^O\right] dt + \left[\sigma P_t^I + \phi^+ \sigma A P_t^O\right] dz_t.$$

Therefore, the future return of the firm is:

$$\begin{aligned} R_t &\equiv \frac{dP_t + \theta_t dt}{P_t} \\ &= \mu dt + \frac{\Omega A P_t^O + \theta_t}{P_t^I + AP_t^O} dt + \frac{\sigma P_t^I + \sigma\phi^+ A P_t^O}{P_t^I + AP_t^O} dz_t, \end{aligned} \tag{O10}$$

where $\Omega = \left(\phi^+ - 1\right)\mu + \phi^+ \left(\phi^+ - 1\right)\sigma^2/2 > 0$.

1.4.2 The Patent Owners' Decision

With the optimal stopping time, we proceed to derive the optimal patent royalty by solving a Cournot Equilibrium. Owner i solves its optimal royalty to maximize its net present profits:

$$\max_{q_i} E_t \left[\frac{M_{\tau^*}}{M_t} (q_i - c_i) \right] = \max_{q_i} E_t \left[e^{-r(\tau^* - t)} (q_i - c_i) \right].$$

Solving the simultaneous maximization problems for all owners, we have Lemma O2.

Lemma O2 *The optimal royalty of owner i is:*

$$q_i^* = \frac{1}{\phi^+ - n} \left[\left(\phi^+ - n \right) c_i + \sum_{j=1}^{n} c_j \right], \text{ and}$$

when $c_i = c_j = c$, the optimum can be simplified to $q_i^* = q_j^* = q^*$ for $j = 1...n$, in which:

$$q^* = \frac{\phi^+}{\phi^+ - n} c. \tag{O11}$$

Similar to the main model, we continue our discussion under the assumption that $c_i = c_j = c$.

1.4.3 Theoretical Predictions

The systematic risk exposure can be derived as:

$$\begin{aligned} \beta_t &\equiv - \frac{E_t \left[\frac{dM_t}{M_t} R_t \right]}{Var_t \left[\frac{dM_t}{M_t} \right]} \\ &= \frac{\sigma}{\kappa} \left[\frac{P_t^I + \phi^+ A P_t^O}{P_t^I + A P_t^O} \right]. \end{aligned} \tag{O12}$$

Taking the first-order derivative of β_t w.r.t. n, we have $d\beta_t / dn < 0$, which is consistent with Proposition 1.

Moreover, differentiating β_t in Equation (O12) with respect to A yields:

$$\frac{d\beta_t}{dA} > 0,$$

and the following proposition.

Proposition O4 *A firm's exposure to systematic risk increases with patent portfolio size.*

The results presented in Table IV support Proposition O4.

From Equation (O10), we can derive:

$$\frac{dE_t\left[R_t\right]}{dn} = \frac{\Omega P_t^I - \theta_t}{\left(P_t^I + AP_t^O\right)^2} \frac{\partial P_t^O}{\partial Q^*} \frac{dQ^*}{dn} dt.$$

Under the condition of $\Omega P_t^I - \theta_t > 0$, we have $dE_t\left[R_t\right]/dn < 0$ (Proposition 2).

Also, differentiating $E_t\left[R_t\right]$ with respect to A yields:

$$\frac{dE_t\left[R_t\right]}{dA} = \frac{\Omega P_t^I - \theta_t}{\left(P_t^I + AP_t^O\right)^2} P_t^O dt > 0,$$

and the following proposition.

Proposition O5 *A firm's expected stock return increases with patent portfolio size.*

Table O.III in the Online Appendix provides supportive evidence to Proposition O5.

2 Online Appendix: Tables

Table O.I

Two-way Portfolio Sorting on Patent Thickets and Patent Portfolio Size

At the end of June of year t from 1982 to 2011, we conduct independent double sorts on patent thicket (PT) and patent portfolio size (CTBE) into 25 groups, based on the quintiles of PT and CTBE at the end of year t – 1. The two variables are defined in Table II. We also construct the "Average" group, which averages the excess returns of the five PT/CTBE groups. Then we construct a high-minus-low (High-Low) portfolio by holding a long (short) position in the top quintile (bottom quintile) PT/CTBE portfolio within each group of the other variable and hold these portfolios over the next twelve months (July of year t to June of year t + 1). We report the monthly returns in excess of the one-month Treasury bill rate (Excess Returns), and their corresponding alphas and betas the CAPM model. MKT denotes the market factor. All returns and alphas are value-weighted and in percentage. The numbers in parentheses denote the standard errors. ***, **, and * indicate significance levels of 1%, 5%, and 10%, respectively. Sample period: July 1982 to June 2012.

Table O.I (continued)

Two-way Portfolio Sorting on Patent Thicket (PT) and Patent Portfolio Size (CTBE)						
PT \ CTBE	**Low**	**2**	**3**	**4**	**High**	**High-Low**
Low	0.67	1.34	1.60	1.32	2.74	2.07***
						(0.54)
2	0.82	0.82	1.33	0.64	1.61	0.78**
						(0.38)
3	0.63	0.69	0.83	1.26	1.32	0.69**
						(0.35)
4	0.49	0.74	0.62	0.80	1.19	0.69*
						(0.38)
High	0.56	0.14	0.57	0.74	0.62	0.06
						(0.28)
High-Low	-0.10	-1.20***	-1.03***	-0.58**	-2.11***	
	(0.21)	(0.31)	(0.32)	(0.27)	(0.57)	

Average across Five Quintiles (High-Low across PT):

Excess Return	**Time-series Mean**	-1.00***
		(0.19)
CAPM	**Alpha**	-0.90***
		(0.19)
	MKT	-0.16***
		(0.04)

Average across Five Quintiles (High-Low across CTBE):

Excess Return	CAPM	
Time-series Mean	**Alpha**	**MKT**
0.86***	0.61***	0.39***
(0.25)	(0.23)	(0.05)